EXCITATION OF ELECTROMAGNETIC WAKE FIELDS BY ONE-DIMENSIONAL ELECTRON BUNCH IN PLASMA IN THE PRESENCE OF CIRCULARLY POLARIZED INTENSE ELECTROMAGNETIC WAVE

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Abstract

The excitation of electromagnetic (EM) wake waves in electron plasma by an one-dimensional bunch of charged particles has been considered in the presence of intense monochromatic circularly polarized electromagnetic (CPEM) pump wave. In the zero state (in the absence of bunch) the interaction of the pump wave with plasma is described by means of Maxwell equations and relativistic nonlinear hydrodynamic equations of cold plasma. The excitation of linear waves by one-dimensional bunch is considered on this background. It is shown that there are three types of solutions of linear equations obtained for induced waves corresponding to three ranges of parameter values of the pump wave, bunch and plasma. In the first range of parameter values the amplitude of transverse components of induced waves is shown to grow as the bunch energy and after some value of the relativistic factor of the bunch to be almost independent of the energy and increase proportional to the intensity and frequency of the pump wave. The dependence of longitudinal component of induced waves on the relativistic factor of the bunch is weak. Its amplitude and wavelength grow as the intensity of pump wave. The second range of parameters is a resonance one. The amplitude of the wave excited by the bunch is a linear function of the distance to the bunch. In the third range of parameter values the longitudinal component of induced fields are localized near the bunch boundaries and are exponentially decreased with the increase in distance from these boundaries. The amplitudes of transverse components of induced waves reach a constant value with the distance from the bunch boundaries.

PACS number(s): 52.35.-g, 52.40.Mj, 52.40.Nk

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1 Introduction

At present the studies on new methods for charged particle acceleration by means of wake fields generated in plasma by laser radiation (BWA (Beat Wave Acceleration), LWFA (Laser Wake Field Acceleration)) and by bunches of relativistic particles (PWFA (Plasma Wake-Field Acceleration)) in flight through plasma are intensively developed (see, e.g., the reviews [1, 2] and references therein). The intensity of acceleration fields (in the order of $10^7 - 10^8 V/cm$), attained by these methods can be used both for the charge acceleration, and for focusing of electron (positron) bunches in order to obtain the beams of high density and to ensure high luminosity in linear colliders of next generation.

The linear theory of wake field generation by rigid bunches of charged particles in boundless and limited plasma was developed in many works (see, e.g., [3-11]). The nonlinear theory of wake field generation by a rigid one-dimensional bunch of finite extent was developed in [12-18]. An important result of this theory is the proof that the wave breaking limit is $E_{\max} = (mv_0\omega_p/e)\left[2(\gamma_0 - 1)\right]^{1/2}$ and it is reached at $n_b/n_0 \lesssim 1/(2 + \gamma_0^{-1})$, where $\omega_p = \left(4\pi n_0 e^2/m\right)^{1/2}$ is the plasma frequency of electrons, n_b and n_0 are the densities of the bunch and plasma electrons, $\gamma_0 = \left(1 - \beta_0^2\right)^{-1/2}$ is the relativistic factor of the bunch, $\beta_0 = v_0/c$. The Dawson [17] wave breaking limit is equal to $E_{\max} \approx 2mv_0\omega_p/e$ when $\gamma_0 \approx 1$ ($\beta_0 \ll 1$). In the linear case $n_b/n_0 \lll 1$, $E_{\max} \simeq 2(mv_0\omega_p/e)(n_b/n_0)$ for arbitrary γ_0.

The one-dimensional relativistic strong waves can be excited in the plasma by wide relativistic bunches of charged particles or intense laser pulses [1, 19], (when $k_p a_0 \gg 1$, where $k_p = \omega_p/v_0$, a_0 are the characteristic transverse sizes of bunches or pulses).

In the present work the effect of the CPEM pump wave with an arbitrary intensity parameter ($A = eE_0/mc\omega_0$, where E_0 and ω_0 are the amplitude and the frequency of the EM wave) on the excitation of EM wake waves by one-dimensional relativistic electron bunch in cold plasma has been studied. Here the rate of electron oscillations in the pump wave may be of the order of light velocity. One can obtain the exact solutions of the Maxwell equations and of nonlinear hydrodynamic equations [18, 20] for CPEM wave interacting with plasma as well as derive the exact dispersion equation for waves propagating in the same direction as pump wave [20, 21]. The parametric instability of the plasma in the presence of CPEM wave has been studied rather well yet in early works (see, e.g., [20, 22, 23] and literature therein). Max and Perkins [22] investigated an aperiodic low frequency instability of plasma in the dipole approximation. The instability of plasma in the presence of strong CPEM wave was considered in [20] and it was shown that at parametric excitation of nonpotential oscillations in plasma by a CPEM the relativistic motion of electrons is essential for the arbitrary value of pump wave amplitude.

This paper is organized as follows. The derivation of equations for the velocity of motion of an electron liquid in plasma and for EM fields excited by an one-dimensional electron bunch in the presence of CPEM wave is given in Sec. II. The interaction of the pump wave with plasma (in the absence of a bunch) is described by Maxwell equations and nonlinear hydrodynamic equations of cold plasma. In this case a spatially homogeneous state of plasma is possible [18, 20]. Then, assuming that this state is weakly perturbed by one-dimensional bunch in the plasma, the equations for induced fields, density and velocity of plasma electrons are obtained using the perturbation theory methods. In Sec. III the general expressions for induced EM fields of one-dimensional bunch with an arbitrary density profile are obtained using the method of Green's functions. There are three ranges of values of plasma, pump wave and bunch parameters, where the properties of the Green's functions and, hence, the behavior of excited waves, are abruptly changed. In Sec.

IV the fields excited in plasma by a bunch with uniform distribution of electron density are considered. For each of the mentioned ranges of parameters the expressions for induced fields components are obtained and corresponding numerical calculations are carried out. A brief summary of the results is given in Sec. V. An alternative method for obtaining the induced fields is discussed in the Appendix.

2 Basic Equations

As an initial system of equations we shall use the Maxwell equations and relativistic hydrodynamic equations of motion of cold electron plasma under assumption that the oscillation velocity of plasma electrons in a CPEM wave much exceeds their thermal velocities and the frequency ω_0 of the CPEM wave is much higher than the frequency of electron-ion collisions:

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} n\mathbf{v} - \frac{4\pi e}{c}\mathbf{u} N_b(\xi), \tag{1}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \qquad \boldsymbol{\nabla}\mathbf{B} = 0, \tag{2}$$

$$\boldsymbol{\nabla}\mathbf{E} = -4\pi e\left(n - n_0\right) - 4\pi e N_b(\xi), \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v} = -\frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} - \frac{\mathbf{v}}{c^2}\left(\mathbf{v}\mathbf{E}\right)\right], \tag{4}$$

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\left(n\mathbf{v}\right) = 0, \tag{5}$$

where n_0 is the unperturbed electron density, $N_b(\xi)$ is the density of one-dimensional electron bunch moving with velocity $\mathbf{u} = u\mathbf{e}_z$, $\xi = z - ut$.

In the CPEM wave propagating along the z axis a spatially homogeneous state of plasma may be established, in which the EM field and velocity of electrons will be determined by expressions [18, 20]

$$\mathbf{E}_0 = E_0\left(\mathbf{e}_x \cos\zeta + \mathbf{e}_y \sin\zeta\right), \qquad E_{0z} = 0, \tag{6}$$

$$\mathbf{B}_0 = \frac{k_0 c}{\omega_0} E_0\left(-\mathbf{e}_x \sin\zeta + \mathbf{e}_y \cos\zeta\right), \qquad B_{0z} = 0, \tag{7}$$

$$\mathbf{v}_e = c\beta_e\left(-\mathbf{e}_x \sin\zeta + \mathbf{e}_y \cos\zeta\right), \qquad v_{ez} = 0, \tag{8}$$

where $\zeta = \omega_0 t - k_0 z$, $k_0 = (\omega_0/c)\sqrt{\varepsilon(\omega_0)}$, $\varepsilon(\omega) = 1 - \omega_L^2/\omega^2$, $\omega_L^2 = \omega_p^2\sqrt{1-\beta_e^2}$, $\beta_e = v_e/c$, $A = eE_0/mc\omega_0$,

$$v_e = c\frac{A}{\sqrt{1+A^2}}, \tag{9}$$

$\omega_p^2 = 4\pi n_0 e^2/m$ is the plasma frequency, c is the velocity of light.

Consider small perturbations of plasma due to the presence of an electron bunch with density of N_b. The linearization implies that the analysis is valid only on condition that the bunch density $N_b \ll n_0$. We shall write all quantities in the form $f = f_0 + f'$, where f_0 are determined by means of Eqs. (6)-(9). The linearization of the system of Eqs. (1)-(5) gives the following set of equations for induced variables f'

$$\nabla \times \mathbf{B}^{'} = \frac{1}{c}\frac{\partial \mathbf{E}^{'}}{\partial t} - \frac{4\pi e}{c}\left(n_0 \mathbf{v}^{'} + n^{'}\mathbf{v}_e\right) - \frac{4\pi e}{c}\mathbf{u}N_b(\xi), \tag{10}$$

$$\nabla \times \mathbf{E}^{'} = -\frac{1}{c}\frac{\partial \mathbf{B}^{'}}{\partial t}, \qquad \nabla \mathbf{B}^{'} = 0, \tag{11}$$

$$\nabla \mathbf{E}^{'} = -4\pi e n^{'} - 4\pi e N_b(\xi), \tag{12}$$

$$\begin{aligned} & \frac{\partial \mathbf{v}^{'}}{\partial t} + \left(\mathbf{v}_e \nabla\right)\mathbf{v}^{'} + \left(\mathbf{v}^{'}\nabla\right)\mathbf{v}_e \\ = & -\frac{e}{m}\sqrt{1-\beta_e^2}\left[\mathbf{E}^{'} + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}^{'} + \frac{1}{c}\mathbf{v}^{'} \times \mathbf{B}_0 - \frac{\mathbf{v}_e}{c^2}\left(\mathbf{v}_e\mathbf{E}^{'} + \mathbf{v}^{'}\mathbf{E}_0\right)\right] + \\ & + \frac{e\left(\mathbf{v}^{'}\mathbf{v}_e\right)}{mc^2\sqrt{1-\beta_e^2}}\left(\mathbf{E}_0 + \frac{1}{c}\mathbf{v}_e \times \mathbf{B}_0\right), \end{aligned} \tag{13}$$

$$\frac{\partial n^{'}}{\partial t} + n_0 \nabla \mathbf{v}^{'} + \left(\mathbf{v}_e \nabla\right) n^{'} = 0. \tag{14}$$

The set of Eqs. (10)-(14) is a system of partial differential equations with periodical coefficients with respect to the variable ζ.

If we pass from x and y components of fields and velocities of plasma electrons to new variables

$$\begin{pmatrix} E^{\pm} \\ B^{\pm} \\ w^{\pm} \end{pmatrix} = \begin{pmatrix} E_x^{'} \pm iE_y^{'} \\ B_x^{'} \pm iB_y^{'} \\ v_x^{'} \pm iv_y^{'} \end{pmatrix} = \begin{pmatrix} \mathcal{E}^{\pm} \\ \mathcal{B}^{\pm} \\ \mathcal{V}^{\pm} \end{pmatrix} e^{i\zeta} \tag{15}$$

this will mean that we convert these into the rotating system of reference of the CPEM wave. Here the set of Eqs. (10)-(14) passes into a system of ordinary inhomogeneous differential equations with constant coefficients

$$\begin{aligned} & \frac{1}{\gamma^2}\frac{\partial^2 \mathcal{E}^{\pm}}{\partial \xi^2} \mp 2i\left(k_0 - \beta\frac{\omega_0}{c}\right)\frac{\partial \mathcal{E}^{\pm}}{\partial \xi} - \left(k_0^2 - \frac{\omega_0^2}{c^2}\right)\mathcal{E}^{\pm} \\ = & \frac{4\pi e n_0}{c}\left(\beta\frac{\partial \mathcal{V}^{\pm}}{\partial \xi} \mp i\frac{\omega_0}{c}\mathcal{V}^{\pm} + \omega_0\beta_e\frac{n^{'}}{n_0} \pm \frac{i\beta_e}{n_0}u\frac{\partial n^{'}}{\partial \xi}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} & \frac{1}{\gamma^2}\frac{\partial^2 \mathcal{B}^{\pm}}{\partial \xi^2} \mp 2i\left(k_0 - \beta\frac{\omega_0}{c}\right)\frac{\partial \mathcal{B}^{\pm}}{\partial \xi} - \left(k_0^2 - \frac{\omega_0^2}{c^2}\right)\mathcal{B}^{\pm} \\ = & \pm\frac{4\pi e i n_0}{c}\left(\frac{\partial \mathcal{V}^{\pm}}{\partial \xi} \mp ik_0\mathcal{V}^{\pm} + k_0 v_e\frac{n^{'}}{n_0} \pm \frac{iv_e}{n_0}\frac{\partial n^{'}}{\partial \xi}\right), \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{\partial^2 \mathcal{V}^{\pm}}{\partial \xi^2} + \frac{\omega_0^2}{u^2}\mathcal{V}^{\pm} = & \frac{e}{mu}\sqrt{1-\beta_e^2} \\ & \left[\left(1 - \frac{\beta_e^2}{2}\right)\left(\frac{\partial \mathcal{E}^{\pm}}{\partial \xi} \pm i\frac{\omega_0}{u}\mathcal{E}^{\pm}\right) + \frac{\beta_e^2}{2}\left(\frac{\partial \mathcal{E}^{\mp}}{\partial \xi} \mp i\frac{\omega_0}{u}\mathcal{E}^{\mp}\right)\right], \end{aligned} \tag{18}$$

$$\frac{\partial E_z^{'}}{\partial \xi} = -4\pi e n^{'} - 4\pi e N_b(\xi), \qquad B_z^{'} = 0, \tag{19}$$

$$\frac{\partial v_z^{'}}{\partial \xi} = \frac{e}{mu}\sqrt{1-\beta_e^2}\left[E_z^{'} - \frac{\beta_e}{2}\left(\mathcal{B}^+ + \mathcal{B}^-\right) + \frac{k_0 E_0}{2\omega_0}\left(\mathcal{V}^+ + \mathcal{V}^-\right)\right], \tag{20}$$

$$n^{'} = n_0 \frac{v_z^{'}}{u}, \tag{21}$$

where $\beta = u/c$. At the derivation of Eqs. (16)-(21) we assumed that all quantities depended on the variable $\xi = z - ut$. One can see from these equations that owing to the presence of CPEM wave in the plasma, the induced fields and velocities of electron liquid motion are parametrically related. Besides the longitudinal component of the induced electric field the right-hand side of Eq. (20) for the induced velocity longitudinal component comprises also the Lorentz force. The second term arises on account of the interaction of unperturbed velocity of plasma electrons with the induced magnetic field. The third term is due to the interaction of induced velocities of electrons with the magnetic field of CPEM wave. As is seen from the expression (20), the coefficient of the second term (β_e) in the square brackets tends to 1/2 and the coefficient of the last term grows with the intensity of the CPEM wave. Below we shall examine the solutions of Eqs. (16)-(21).

3 Calculations of Green's Functions

To solve the system of Eqs. (16)-(21) one can obtain the equations for each of variables $n^{'}$, $E_z^{'}$, $\mathcal{V}^{\pm}$, $\mathcal{B}^{\pm}$, $\mathcal{E}^{\pm}$. Such an approach to determination of induced fields and velocities of electrons has its advantages and is discussed in short in the Appendix. Here we shall solve the system of Eqs. (16)-(21) by expansion of induced quantities in Fourier integral in variable ξ. Then, after some transformations we find

$$\begin{pmatrix} E_z^{'}(\xi) \\ \mathcal{E}^+(\xi) \\ \mathcal{B}^+(\xi) \end{pmatrix} = \int_{-\infty}^{+\infty} d\xi^{'} N_b(\xi^{'}) \begin{pmatrix} G_z^{(e)}(\xi^{'} - \xi) \\ G_\perp^{(e)}(\xi^{'} - \xi) \\ G_\perp^{(m)}(\xi^{'} - \xi) \end{pmatrix}, \tag{22}$$

where $G_z^{(e)}, G_\perp^{(e)}, G_\perp^{(m)}$ are the Green's functions respectively for quantities $E_z^{'}$, $\mathcal{E}^+$ and $\mathcal{B}^+$ that are as follows:

$$G_z^{(e)}(s) = -2ie\int_{-\infty}^{+\infty} \frac{dk}{k}\frac{D_1(k,\omega)}{D(k,\omega)} e^{iks}, \tag{23}$$

$$G_\perp^{(e)}(s) = -\frac{2e\beta_e\omega_L^2}{c}\int_{-\infty}^{+\infty} dk(ku+\omega_0)\frac{R_{-1}(k,\omega)}{D(k,\omega)} e^{iks}, \tag{24}$$

$$G_\perp^{(m)}(s) = -2ie\beta_e\omega_L^2\int_{-\infty}^{+\infty} dk(k+k_0)\frac{R_{-1}(k,\omega)}{D(k,\omega)} e^{iks}, \tag{25}$$

where $\omega = ku$, $\gamma^{-2} = 1-\beta^2$. Here we made the following notations:

$$R_{\pm 1}(k,\omega) = (k\pm k_0)^2 - \frac{(\omega\pm\omega_0)^2}{c^2}\varepsilon(\omega\pm\omega_0), \tag{26}$$

$$D_1(k,\omega) = \omega^2 R_1(k,\omega)R_{-1}(k,\omega) + \frac{\beta_e^2\omega_L^2}{2}\left(k^2 - \frac{\omega^2}{c^2}\right)\left[R_1(k,\omega) + R_{-1}(k,\omega)\right], \tag{27}$$

$$D(k,\omega) = \omega^2\varepsilon(\omega) R_1(k,\omega)R_{-1}(k,\omega) + \frac{\beta_e^2\omega_L^2}{2}\left[k^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right]\left[R_1(k,\omega) + R_{-1}(k,\omega)\right]. \tag{28}$$

From expressions (15) one can find the transverse components of induced electric and magnetic fields, as well as the velocities of plasma electron motion by taking the real or imaginary parts from complex quantities $\mathcal{E}^+$, $\mathcal{B}^+$ and w^+. As a result we have

$$\begin{pmatrix} E_x^{'}(z,t) \\ B_x^{'}(z,t) \end{pmatrix} = \begin{pmatrix} E_r(\xi) \\ B_r(\xi) \end{pmatrix} \cos(\zeta) - \begin{pmatrix} E_i(\xi) \\ B_i(\xi) \end{pmatrix} \sin(\zeta), \tag{29}$$

$$\begin{pmatrix} E_y^{'}(z,t) \\ B_y^{'}(z,t) \end{pmatrix} = \begin{pmatrix} E_r(\xi) \\ B_r(\xi) \end{pmatrix} \sin(\zeta) + \begin{pmatrix} E_i(\xi) \\ B_i(\xi) \end{pmatrix} \cos(\zeta), \tag{30}$$

where

$$\begin{pmatrix} E_r(\xi) \\ B_r(\xi) \end{pmatrix} = \mathrm{Re} \begin{pmatrix} \mathcal{E}^+(\xi) \\ \mathcal{B}^+(\xi) \end{pmatrix}, \qquad \begin{pmatrix} E_i(\xi) \\ B_i(\xi) \end{pmatrix} = \mathrm{Im} \begin{pmatrix} \mathcal{E}^+(\xi) \\ \mathcal{B}^+(\xi) \end{pmatrix}. \tag{31}$$

Thus, one can see from expressions (29) and (30) that the transverse components of induced fields describe the modulated oscillations in plasma. E.g., one can represent $E_y^{'}$ in the form

$$E_y^{'}(z,t) = E_{\perp 0}(\xi) \sin\left(\zeta + \psi_0(\xi)\right). \tag{32}$$

Here $E_{\perp 0}(\xi) = \sqrt{E_r^2(\xi) + E_i^2(\xi)}$ is the amplitude and $\psi_0(\xi) = \arctan[E_i(\xi)/E_r(\xi)]$ is the phase displacement of oscillations (other induced quantities may be similarly written in the form of (32)). In the system of reference of bunch rest the expression (32) describes the transverse harmonic wave. In an arbitrary reference system the expression (32) describes a modulated transverse wave, the profile of which is given by the function $E_{\perp 0}(\xi)$. Note also that in the absence of CPEM wave ($\beta_e = 0$) all transverse components of induced quantities turn to zero, and Eq. (19) and its solution (the first expression in Eq. (22) together with expressions (27) and (28)) turn into the known expressions for linear one-dimensional fields.

Now calculate the Green's functions that are determined by expressions (23)-(25). The poles of integrals in the expressions (23)-(25) are determined by the dispersion equation $D(k,\omega) = 0$, that has been comprehensively studied for an arbitrary case in Ref. [20] (i.e., without the $\check{\mathrm{C}}$herenkov condition $\omega = ku$ to be imposed). In the absence of a CPEM wave ($\beta_e = 0$) we obtain dispersion equations for ordinary plasma and transverse (EM) waves from the expression (28) with $\omega = \omega_p$ and $\omega^2 = \omega_p^2 + k^2c^2$ respectively. In the presence of a pump wave ($\beta_e \neq 0$) there arise coupling waves in the plasma, the growth increment of which for small amplitudes of the CPEM wave ($\beta_e \ll 1$) linearly increases as β_e. Thus, the parametric instability of coupling waves persists down to the value of $\beta_e = 0$. However, in the case of non-dense plasma ($\omega_0 \lesssim 10^{13}\,\mathrm{sec}^{-1}$ or $n_0 \lesssim 10^{17}\mathrm{cm}^{-3}$) and for optical values of ω_0 ($\omega_0 \sim 10^{15}\,\mathrm{sec}^{-1}$), the time of relativistic bunch interaction is much less than the time of parametric instability development [20] and here we shall not make any allowance for their effect on the excitation of waves in plasma.

In case when the $\check{\mathrm{C}}$herenkov condition is approached ($\omega = ku$) we obtain from expression (28) the following dispersion equation

$$\left(k^2 - \frac{\omega_L^2}{u^2}\right)\left[k^2 - 4\gamma^4\left(k_0 - \beta\frac{\omega_0}{c}\right)^2\right] + \frac{\beta_e^2\omega_L^2}{u^2}\left(k^2 + \gamma^2\frac{\omega_L^2}{c^2}\right) = 0. \tag{33}$$

If we introduce a dimensionless wave vector λ that is related to k by means of expression $k = (\omega_L/u)\lambda$, then the solution of Eq. (3) will take on the form

$$\lambda_{\pm}^2 = \frac{1}{2a^4} + 2\beta^2\gamma^4 F^2 \pm \sqrt{\left(\frac{1}{2a^4} + 2\beta^2\gamma^4 F^2\right)^2 - \beta^2\gamma^2\left(\frac{a^4-1}{a^4} + 4\gamma^2 F^2\right)}, \quad (34)$$

where

$$F = \sqrt{a^2\Delta^2 - 1} - \beta a \Delta, \quad (35)$$

$\Delta = \omega_0/\omega_p$, $a^2 = \sqrt{1 + A^2}$.

The behavior of solutions of Eq. (33) and, consequently, the nature of induced fields and of velocity of plasma electrons strongly depend on the sign of radicand in the Eq. (34). In the following we shall identify the regions where the radicand is positive, equal to zero or negative as the I^{st}, $\mathrm{II}^{\mathrm{nd}}$, and $\mathrm{III}^{\mathrm{rd}}$ regions respectively. First, we shall determine the boundaries of the region where this radicand turns to zero. If the radicand is made equal to zero one finds for the boundary of the above-mentioned region

$$\Delta = \frac{\gamma}{a}\left(\sqrt{1 + \gamma^2 F_+^2} \pm \beta\gamma F_+\right), \quad (36)$$

when $a > 1$, and

$$\Delta = \frac{\gamma}{a}\left(\sqrt{1 + \gamma^2 F_-^2} \pm \beta\gamma F_-\right), \quad (37)$$

when $1 < a < a_0(\gamma)$, where

$$F_{\pm} = \sqrt{\frac{2 - 1/a^4 \pm 2\gamma\sqrt{1 - 1/a^4}}{4\gamma^2\left(\gamma^2 - 1\right)}}, \quad (38)$$

$$a_0(\gamma) = \left\{\frac{\gamma + \sqrt{\gamma^2 - 1}}{2\sqrt{\gamma^2 - 1}}\right\}^{1/4}. \quad (39)$$

The Eqs. (36)-(39) were obtained on the assumption that $\gamma > \gamma_1 \cong 1.45$, where γ_1 is a real positive root of equation $2\gamma^2(\gamma^2 - 2) = \gamma - 1$ that satisfies the condition $\gamma > 1$.

For small pump wave amplitude ($a - 1 \ll 1$) the values of functions given by the first expressions (with plus sign) and those of functions given by the second expressions (with minus sign) of Eqs. (36) and (37) coincide. We denote these values respectively as $\Delta_{\pm}$ and from Eqs. (36) and (37) we can obtain for them the following expressions:

$$\Delta_{\pm} = \gamma\sqrt{1 + \frac{1}{4(\gamma^2 - 1)}} \pm \frac{1}{2}. \quad (40)$$

In Figs. 1, 2, and 3 the regions I, II and III (the lines coincide with the region II) are given for values of $\gamma = 1.5$, $\gamma = 20$ and $\gamma = 100$ respectively. The lines in these figures are closed at the infinity (i.e., when $a \to \infty$). For modern lasers with intensities $I_L \lesssim 10^{20}\mathrm{W/cm}^2$ we have the following restriction on the parameter a: $a \lesssim 3$ for values of frequency of about $\omega_0 \simeq 3 \times 10^{15}\mathrm{sec}^{-1}$. So, as is seen from Figs. 1, 2 and 3 and Eqs. (36) and (37), for values of parameters $\omega_0 \simeq 10^{15}\mathrm{sec}^{-1}$, $n_0 \lesssim 10^{17}\mathrm{cm}^{-3}$ ($\omega_p \lesssim 10^{13}\mathrm{sec}^{-1}$) the solutions of Eq. (35) are in the region I for wide spread of the values of γ (up to the values of $\gamma \lesssim (\omega_0/\omega_p)a$ and higher). The solutions of Eq. (33) will be in the regions III or II if the condition $\gamma \sim a\Delta = (\omega_0/\omega_p)a \gtrsim 100$ is observed (the ultrarelativistic bunch).

Now we shall calculate the Green's function for each of the regions in separate.

3.1 Region I

In this case the roots of Eq. (33) are real and lie within the upper half plane of the complex variable k. Integrating the expressions (23)-(25) over the variable k we find

$$\begin{aligned} G_z^{(e)}(s) &= \frac{4\pi e\theta(s)}{\lambda_+^2-\lambda_-^2}\left[\left(\lambda_+^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\cos\left(\frac{\lambda_+}{a}k_ps\right)-\right. \\ &\left.-\left(\lambda_-^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\cos\left(\frac{\lambda_-}{a}k_ps\right)\right], \end{aligned} \tag{41}$$

$$\begin{aligned} G_\perp^{(e)}(s) &= \pi ie\frac{\sqrt{a^4-1}}{a}\frac{\Delta F}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]+ \\ &+\frac{4\pi e\beta\gamma^2}{\lambda_+^2-\lambda_-^2}\frac{\sqrt{a^4-1}}{a^2}\theta(s)\left\{\left(a\Delta-2\beta\gamma^2F\right)\left[\frac{1}{\lambda_+}\sin\left(\frac{\lambda_+}{a}k_ps\right)-\frac{1}{\lambda_-}\sin\left(\frac{\lambda_-}{a}k_ps\right)\right]+\right. \\ &\left.+i\left(1-\frac{2\beta\gamma^2a\Delta F}{\lambda_-^2}\right)\cos\left(\frac{\lambda_-}{a}k_ps\right)-i\left(1-\frac{2\beta\gamma^2a\Delta F}{\lambda_+^2}\right)\cos\left(\frac{\lambda_+}{a}k_ps\right)\right\}, \end{aligned} \tag{42}$$

$$\begin{aligned} G_\perp^{(m)}(s) &= -\pi e\frac{\sqrt{a^4-1}}{a^2}\frac{FF_0}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]+\frac{4\pi e\gamma^2}{\lambda_+^2-\lambda_-^2}\frac{\sqrt{a^4-1}}{a^2}\theta(s)\times \\ &\times\left\{\left(1-\frac{2\beta^2\gamma^2FF_0}{\lambda_+^2}\right)\cos\left(\frac{\lambda_+}{a}k_ps\right)-\left(1-\frac{2\beta^2\gamma^2FF_0}{\lambda_-^2}\right)\cos\left(\frac{\lambda_-}{a}k_ps\right)-\right. \\ &\left.-i\beta\left(2\gamma^2F-F_0\right)\left[\frac{1}{\lambda_+}\sin\left(\frac{\lambda_+}{a}k_ps\right)-\frac{1}{\lambda_-}\sin\left(\frac{\lambda_-}{a}k_ps\right)\right]\right\}, \end{aligned} \tag{43}$$

where $k_p=\omega_p/u$, $F_0=\sqrt{a^2\Delta^2-1}$, $\theta(s)$ is the Heavyside function. It is seen from Eqs. (41)-(43) that the bunch excites in plasma the oscillations of two types with frequencies $\omega_L\lambda_+$ and $\omega_L\lambda_-$. In the $a=1$ limit (the CPEM wave is absent) $G_\perp^{(e)}$ and $G_\perp^{(m)}$ turn to zero and for $G_z^{(e)}$ we obtain the expression

$$G_z^{(e)}(s)=4\pi e\theta(s)\cos(k_ps), \tag{44}$$

that coincides with those given in Refs. [24, 25].

3.2 Region II

In the second region the radicand in Eq. (34) turns zero. In this case $\lambda_+^2=\lambda_-^2=\lambda_0^2$,

$$\lambda_0^2=\frac{1}{2a^4}+2\beta^2\gamma^4F^2=1\pm\gamma\frac{\sqrt{a^4-1}}{a^2}>0, \tag{45}$$

where the sign "+" corresponds to the values of parameters $1<a$, $\gamma>\gamma_1$ (see the Eq. (36)). The sign "–" corresponds to the values of parameters $1<a<a_0(\gamma)$, $\gamma>\gamma_1$ (see the Eq. (37)). The poles $\pm\lambda_0$ in this region are multiple ones and lie in the upper complex plane k. The calculation of integrals in Eqs. (23)-(25) gives

$$G_z^{(e)}(s)=4\pi e\theta(s)\left[\cos\left(\frac{\lambda_0}{a}k_ps\right)-\frac{k_ps}{2a\lambda_0}\left(\lambda_0^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\sin\left(\frac{\lambda_0}{a}k_ps\right)\right], \tag{46}$$

$$
\begin{aligned}
G_{\perp}^{(e)}(s) &= \pi i e\frac{\sqrt{a^4-1}}{a}\frac{\Delta F}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]- \\
&-\frac{2\pi e\beta\gamma^2}{\lambda_0^3}\frac{\sqrt{a^4-1}}{a^2}\theta(s)\left\{\left[a\Delta-2\beta\gamma^2 F+i\left(2\beta\gamma^2\Delta F-\frac{\lambda_0^2}{a}\right)(k_p s)\right]\sin\left(\frac{\lambda_0}{a}k_p s\right)-\right. \\
&\left.-\left[\left(a\Delta-2\beta\gamma^2 F\right)\frac{\lambda_0}{a}(k_p s)-\frac{4i\beta\gamma^2 a\Delta F}{\lambda_0}\right]\cos\left(\frac{\lambda_0}{a}k_p s\right)\right\},
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
G_{\perp}^{(m)}(s) &= -\pi e\frac{\sqrt{a^4-1}}{a^2}\frac{FF_0}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]+\frac{2\pi e\gamma^2}{\lambda_0^3}\frac{\sqrt{a^4-1}}{a^2}\theta(s) \\
&\times\left\{\left[\frac{4\beta^2\gamma^2 FF_0}{\lambda_0}-i\beta\left(2\gamma^2F-F_0\right)\frac{\lambda_0}{a}(k_p s)\right]\cos\left(\frac{\lambda_0}{a}k_p s\right)+\right. \\
&\left.+\left[\frac{k_p s}{a}\left(2\beta^2\gamma^2FF_0-\lambda_0^2\right)+i\beta\left(2\gamma^2F-F_0\right)\right]\sin\left(\frac{\lambda_0}{a}k_p s\right)\right\}.
\end{aligned} \tag{48}
$$

Note, that one could obtain the Eqs. (46)-(48) from expressions (41)-(43) by tending $\lambda_+ \to \lambda_- \to \lambda_0$ in the latter. The uncertainty that arises here may be eliminated by using the L'Hopital rule.

Unlike the Eqs. (41)-(43) the Eqs. (46)-(48) contain oscillating terms, the amplitudes of which are linear functions of s. We shall see below that such a dependence of Green's functions leads to an exponential growth of the induced fields as a function of coordinates.

3.3 Region III

In this case the roots of dispersion Eq. (33) are complex quantities and are distributed over the complex plane λ symmetrical with respect to the origin of coordinates. We shall denote the roots in the upper half plane as $\lambda_\pm = \pm\alpha + i\delta$ (the roots in the lower complex half plane will be $\pm\alpha - i\delta$), where

$$
\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \sqrt{\frac{\sqrt{P_1^2+P_2^2}\pm P_1}{2}}, \tag{49}
$$

$$
P_1 = \frac{1}{2a^4} + 2\beta^2\gamma^4F^2, \tag{50}
$$

$$
P_2 = \sqrt{\beta^2\gamma^2\left(\frac{a^4-1}{a^4}+4\gamma^2F^2\right)-P_1^2}. \tag{51}
$$

After calculation of the integrals over k in Eqs. (23)-(25) we have the following expressions for the Green's functions:

$$
\begin{aligned}
G_z^{(e)}(s) &= \pi e\left[\theta(s)-\theta(-s)\right]\exp\left(-\frac{\delta}{a}k_p|s|\right) \\
&\times\left[\frac{1}{\alpha\delta}\left(\alpha^2-\delta^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\sin\left(\frac{\alpha}{a}k_p|s|\right)+2\cos\left(\frac{\alpha}{a}k_p s\right)\right],
\end{aligned} \tag{52}
$$

$$
G_{\perp}^{(e)}(s) = \pi i e\frac{\sqrt{a^4-1}}{a}\frac{\Delta F}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]-\frac{\pi e\beta\gamma^2}{\alpha\delta\left(\alpha^2+\delta^2\right)}\frac{\sqrt{a^4-1}}{a^2}\exp\left(-\frac{\delta}{a}k_p|s|\right) \tag{53}
$$

$$\times\left\{\alpha\left[\left(a\Delta-2\beta\gamma^2F\right)\frac{|s|}{s}+\frac{4i\delta\beta\gamma^2a\Delta F}{\alpha^2+\delta^2}\right]\cos\left(\frac{\alpha}{a}k_ps\right)+\right.$$
$$\left.+\left[\delta\left(a\Delta-2\beta\gamma^2F\right)\frac{|s|}{s}+i\frac{\left(\alpha^2+\delta^2\right)^2-2\beta\gamma^2a\Delta F\left(\alpha^2-\delta^2\right)}{\alpha^2+\delta^2}\right]\sin\left(\frac{\alpha}{a}k_p|s|\right)\right\},$$

$$G_{\perp}^{(m)}(s) = -\pi e\frac{\sqrt{a^4-1}}{a^2}\frac{FF_0}{F^2+\frac{a^4-1}{4a^4}\gamma^{-2}}\left[\theta(s)-\theta(-s)\right]+\frac{\pi e\gamma^2}{\alpha\delta\left(\alpha^2+\delta^2\right)}\frac{\sqrt{a^4-1}}{a^2}\exp\left(-\frac{\delta}{a}k_p|s|\right) \tag{54}$$
$$\times\left\{\alpha\beta\left[\frac{4\delta\beta\gamma^2FF_0}{\alpha^2+\delta^2}\frac{|s|}{s}+i\left(2\gamma^2F-F_0\right)\right]\cos\left(\frac{\alpha}{a}k_ps\right)+\right.$$
$$\left.+\left[\frac{\left(\alpha^2+\delta^2\right)^2-2\beta^2\gamma^2FF_0\left(\alpha^2-\delta^2\right)}{\alpha^2+\delta^2}\frac{|s|}{s}+i\beta\delta\left(2\gamma^2F-F_0\right)\right]\sin\left(\frac{\alpha}{a}k_p|s|\right)\right\}.$$

It follows from Eqs. (52)-(54) that in the region III the Green's functions contain terms that oscillate with exponentially decreasing amplitude.

4 Induced Fields for Specific Choice of $N_b(\xi)$

In this section we shall calculate and investigate the induced fields for a specific profile of electron bunch density. We shall assume that the electrons are homogeneously distributed in the bunch with the density n_b ($n_b \ll n_0$), and the length of bunch is d, i.e.,

$$N_b(\xi) = n_b\left[\theta(\xi)-\theta(\xi-d)\right]. \tag{55}$$

As was noted in Sec. III, the values of induced fields strongly depend on the fact, in which of three mentioned regions lie the values of the discriminant of Eq. (33). The induced fields in each region (I, II or III) of the values of bunch, plasma and pump wave parameters will be considered separately.

4.1 Region I

In this case the Green's functions are determined by the Eqs. (41)-(43). The substitution of these expressions and Eq. (55) into the Eq. (22) will result in the obtaining of following expressions for the induced fields ahead of ($\xi > d$), inside ($0 < \xi < d$) and behind ($\xi < 0$) the bunch:

$\xi > d$;

$$E_z^{'}(\xi) = 0, \tag{56}$$

$$\mathcal{E}^+(\xi) = -i\widetilde{E}_0\frac{n_b}{n_0}(k_pd)\frac{\sqrt{a^4-1}}{a}\frac{\beta\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}, \tag{57}$$

$$\mathcal{B}^+(\xi) = \widetilde{E}_0\frac{n_b}{n_0}\left(k_pd\right)\frac{\sqrt{a^4-1}}{a^2}\frac{\beta FF_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}. \tag{58}$$

$0 \le \xi \le d$;

$$E_z^{'}(\xi) = \widetilde{E}_0\frac{n_b}{n_0}\frac{a\beta}{\lambda_+^2-\lambda_-^2}\left\{\frac{1}{\lambda_+}\left(\lambda_+^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)\right. \tag{59}$$
$$\left.-\frac{1}{\lambda_-}\left(\lambda_-^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)\right\},$$

$$\begin{aligned}\mathcal{E}^{+}(\xi) &= i\widetilde{E}_0\frac{n_b}{n_0}\frac{\beta\sqrt{a^4-1}}{a}\left\{\frac{\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}k_p(d-2\xi)-\right. \\ &-\frac{\beta\gamma^2}{\lambda_+^2-\lambda_-^2}\left[\frac{\lambda_+^2-2\beta\gamma^2 a\Delta F}{\lambda_+^3}\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)-\frac{\lambda_-^2-2\beta\gamma^2 a\Delta F}{\lambda_-^3}\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)+\right. \\ &\left.\left.+i\left(a\Delta-2\beta\gamma^2F\right)\left[\frac{1}{\lambda_+^2}\left(1-\cos\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)\right)-\frac{1}{\lambda_-^2}\left(1-\cos\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)\right)\right]\right]\right\},\end{aligned} \tag{60}$$

$$\begin{aligned}\mathcal{B}^{+}(\xi) &= \widetilde{E}_0\frac{n_b}{n_0}\frac{\beta\sqrt{a^4-1}}{a^2}\left\{-\frac{FF_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}k_p(d-2\xi)+\right. \\ &+\frac{a\gamma^2}{\lambda_+^2-\lambda_-^2}\left[\frac{\lambda_+^2-2\beta^2\gamma^2FF_0}{\lambda_+^3}\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)-\frac{\lambda_-^2-2\beta^2\gamma^2FF_0}{\lambda_-^3}\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)-\right. \\ &\left.\left.-i\beta\left(2\gamma^2F-F_0\right)\left[\frac{1}{\lambda_+^2}\left(1-\cos\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)\right)-\frac{1}{\lambda_-^2}\left(1-\cos\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)\right)\right]\right]\right\}.\end{aligned} \tag{61}$$

$\xi < 0$;

$$\begin{aligned}E_z^{'}(\xi) &= \widetilde{E}_0\frac{n_b}{n_0}\frac{a\beta}{\lambda_+^2-\lambda_-^2}\left\{\frac{1}{\lambda_+}\left(\lambda_+^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\left[\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_+}{a}k_p\xi\right)\right]-\right. \\ &\left.-\frac{1}{\lambda_-}\left(\lambda_-^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\left[\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_-}{a}k_p\xi\right)\right]\right\},\end{aligned} \tag{62}$$

$$\begin{aligned}\mathcal{E}^{+}(\xi) &= i\widetilde{E}_0\frac{n_b}{n_0}\frac{\beta\sqrt{a^4-1}}{a}\left\{\frac{\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}(k_pd)-\right. \\ &-\frac{\beta\gamma^2}{\lambda_+^2-\lambda_-^2}\left[\frac{\lambda_+^2-2\beta\gamma^2 a\Delta F}{\lambda_+^3}\left(\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_+}{a}k_p\xi\right)\right)-\right. \\ &-\frac{\lambda_-^2-2\beta\gamma^2 a\Delta F}{\lambda_-^3}\left(\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_-}{a}k_p\xi\right)\right)+ \\ &+i\left(a\Delta-2\beta\gamma^2F\right)\left[\frac{1}{\lambda_+^2}\left(\cos\left(\frac{\lambda_+}{a}k_p\xi\right)-\cos\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)\right)-\right. \\ &\left.\left.\left.-\frac{1}{\lambda_-^2}\left(\cos\left(\frac{\lambda_-}{a}k_p\xi\right)-\cos\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)\right)\right]\right]\right\},\end{aligned} \tag{63}$$

$$\begin{aligned}\mathcal{B}^{+}(\xi) &= \widetilde{E}_0\frac{n_b}{n_0}\frac{\beta\sqrt{a^4-1}}{a^2}\left\{-\frac{FF_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}(k_pd)+\right. \\ &+\frac{a\gamma^2}{\lambda_+^2-\lambda_-^2}\left[\frac{\lambda_+^2-2\beta^2\gamma^2FF_0}{\lambda_+^3}\left(\sin\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_+}{a}k_p\xi\right)\right)-\right. \\ &-\frac{\lambda_-^2-2\beta^2\gamma^2FF_0}{\lambda_-^3}\left(\sin\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)+\sin\left(\frac{\lambda_-}{a}k_p\xi\right)\right)- \\ &-i\beta\left(2\gamma^2F-F_0\right)\left[\frac{1}{\lambda_+^2}\left(\cos\left(\frac{\lambda_+}{a}k_p\xi\right)-\cos\left(\frac{\lambda_+}{a}k_p(d-\xi)\right)\right)-\right. \\ &\left.\left.\left.-\frac{1}{\lambda_-^2}\left(\cos\left(\frac{\lambda_-}{a}k_p\xi\right)-\cos\left(\frac{\lambda_-}{a}k_p(d-\xi)\right)\right)\right]\right]\right\}.\end{aligned} \tag{64}$$

Here we have adopted the notation $\widetilde{E}_0 = mc\omega_p/e$.

4.2 Region II

In this range of values of bunch, plasma and pump wave parameters the induced fields ahead of the bunch ($\xi > d$) coincide with the values of induced EM fields in the region I and are given by the expressions (56)-(58). The induced fields inside and behind the bunch are

$0 \le \xi \le d$;

$$E_z^{'}(\xi) = \widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta}{2\lambda_0^2} \left[a \frac{3\lambda_0^2 - 1}{\lambda_0} \sin\left(\frac{\lambda_0}{a} k_p(d-\xi)\right) + k_p(d-\xi)(1-\lambda_0^2) \cos\left(\frac{\lambda_0}{a} k_p(d-\xi)\right)\right], \tag{65}$$

$$\begin{aligned} \mathcal{E}^{+}(\xi) &= i\widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta\sqrt{a^4-1}}{a} \left\{ \frac{\Delta F}{4F^2 + \frac{a^4-1}{a^4}\gamma^{-2}} k_p(d-2\xi) + \frac{i\beta\gamma^2}{\lambda_0^4}\left(a\Delta - 2\beta\gamma^2 F\right) - \right. \\ &- \frac{\beta\gamma^2}{2\lambda_0^5} \left[\left[6\beta\gamma^2 a\Delta F - \lambda_0^2 + i\lambda_0^2\left(a\Delta - 2\beta\gamma^2 F\right) \frac{k_p(d-\xi)}{a}\right] \sin\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + \right. \\ &\left.\left. + \lambda_0 \left[\left(\lambda_0^2 - 2\beta\gamma^2 a\Delta F\right) \frac{k_p(d-\xi)}{a} + 2i\left(a\Delta - 2\beta\gamma^2 F\right)\right] \cos\left(\frac{\lambda_0}{a}k_p(d-\xi)\right)\right]\right\}, \end{aligned} \tag{66}$$

$$\begin{aligned} \mathcal{B}^{+}(\xi) &= \widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta\sqrt{a^4-1}}{a^2} \left\{ - \frac{F F_0}{4F^2 + \frac{a^4-1}{a^4}\gamma^{-2}} k_p(d-2\xi) + \frac{a\gamma^2}{2\lambda_0^5}\left[2i\beta\lambda_0\left(2\gamma^2 F - F_0\right) + \right.\right. \\ &+ \left[6\beta^2\gamma^2 F F_0 - \lambda_0^2 - i\beta\lambda_0^2\left(2\gamma^2 F - F_0\right) \frac{k_p(d-\xi)}{a}\right] \sin\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + \\ &\left.\left. + \lambda_0 \left[\left(\lambda_0^2 - 2\beta^2\gamma^2 F F_0\right) \frac{k_p(d-\xi)}{a} - 2i\beta\left(2\gamma^2 F - F_0\right)\right] \cos\left(\frac{\lambda_0}{a}k_p(d-\xi)\right)\right]\right\}. \end{aligned} \tag{67}$$

$\xi < 0$;

$$\begin{aligned} E_z^{'}(\xi) &= \widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta}{2\lambda_0^2} \left\{ a \frac{3\lambda_0^2 - 1}{\lambda_0} \left[\sin\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + \sin\left(\frac{\lambda_0}{a}k_p\xi\right)\right] + \right. \\ &\left. + (1-\lambda_0^2)\left[k_p(d-\xi)\cos\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + (k_p\xi)\cos\left(\frac{\lambda_0}{a}k_p\xi\right)\right]\right\}, \end{aligned} \tag{68}$$

$$\begin{aligned} \mathcal{E}^{+}(\xi) &= i\widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta\sqrt{a^4-1}}{a} \left\{ \frac{\Delta F k_p d}{4F^2 + \frac{a^4-1}{a^4}\gamma^{-2}} - \right. \\ &- \frac{\beta\gamma^2}{2\lambda_0^5} \left[\left[6\beta\gamma^2 a\Delta F - \lambda_0^2 + i\lambda_0^2\left(a\Delta - 2\beta\gamma^2 F\right) \frac{k_p(d-\xi)}{a}\right] \sin\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + \right. \\ &+ \left[6\beta\gamma^2 a\Delta F - \lambda_0^2 - i\lambda_0^2\left(a\Delta - 2\beta\gamma^2 F\right) \frac{k_p\xi}{a}\right] \sin\left(\frac{\lambda_0}{a}k_p\xi\right) + \\ &+ \lambda_0 \left[\left(\lambda_0^2 - 2\beta\gamma^2 a\Delta F\right) \frac{k_p(d-\xi)}{a} + 2i\left(a\Delta - 2\beta\gamma^2 F\right)\right] \cos\left(\frac{\lambda_0}{a}k_p(d-\xi)\right) + \\ &\left.\left. + \lambda_0 \left[\left(\lambda_0^2 - 2\beta\gamma^2 a\Delta F\right) \frac{k_p\xi}{a} - 2i\left(a\Delta - 2\beta\gamma^2 F\right)\right] \cos\left(\frac{\lambda_0}{a}k_p\xi\right)\right]\right\}, \end{aligned} \tag{69}$$

$$\mathcal{B}^{+}(\xi) = \widetilde{E}_0 \frac{n_b}{n_0} \frac{\beta\sqrt{a^4-1}}{a^2} \left\{ - \frac{F F_0}{4F^2 + \frac{a^4-1}{a^4}\gamma^{-2}} (k_p d) + \right. \tag{70}$$

$$+\frac{a\gamma^2}{2\lambda_0^5}\left[\left[6\beta^2\gamma^2 FF_0-\lambda_0^2-i\beta\lambda_0^2\left(2\gamma^2F-F_0\right)\frac{k_p(d-\xi)}{a}\right]\sin\left(\frac{\lambda_0}{a}k_p(d-\xi)\right)+\right.$$
$$+\left[6\beta^2\gamma^2 FF_0-\lambda_0^2+i\beta\lambda_0^2\left(2\gamma^2F-F_0\right)\frac{k_p\xi}{a}\right]\sin\left(\frac{\lambda_0}{a}k_p\xi\right)+$$
$$+\lambda_0\left[\left(\lambda_0^2-2\beta^2\gamma^2FF_0\right)\frac{k_p(d-\xi)}{a}-2i\beta\left(2\gamma^2F-F_0\right)\right]\cos\left(\frac{\lambda_0}{a}k_p(d-\xi)\right)+$$
$$\left.+\lambda_0\left[\left(\lambda_0^2-2\beta^2\gamma^2FF_0\right)\frac{k_p\xi}{a}+2i\beta\left(2\gamma^2F-F_0\right)\right]\cos\left(\frac{\lambda_0}{a}k_p\xi\right)\right]\Bigg\},$$

where λ_0 is determined by the Eq. (45).

4.3 Region III

In this region the Green's functions are determined by Eqs. (52)-(54). The expression for longitudinal electric fields over the space ($-\infty<\xi<+\infty$) is obtained after the substitution of Eqs. (52)-(54) into the Eq. (22) and appropriate integrations:

$$\begin{aligned}E_z^{'}(\xi) &= \widetilde{E}_0\frac{n_b}{n_0}\frac{a\beta}{4\left(\alpha^2+\delta^2\right)^2}\left\{-\frac{1}{\alpha}\left(\alpha^2+\delta^2-\frac{a^4-1}{a^4}+4\beta^2\gamma^4F^2\right)\times\right. &(71)\\ &\times\left[\exp\left(-\frac{\delta}{a}k_p|\xi|\right)\sin\left(\frac{\alpha}{a}k_p|\xi|\right)-\exp\left(-\frac{\delta}{a}k_p|\xi-d|\right)\sin\left(\frac{\alpha}{a}k_p|\xi-d|\right)\right]+\\ &+\frac{1}{\delta}\left(\alpha^2+\delta^2+\frac{a^4-1}{a^4}-4\beta^2\gamma^4F^2\right)\times\\ &\left.\times\left[\exp\left(-\frac{\delta}{a}k_p|\xi|\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p|\xi-d|\right)\cos\left(\frac{\alpha}{a}k_p(\xi-d)\right)\right]\right\}.\end{aligned}$$

The expressions for induced transverse electromagnetic fields ahead of ($\xi>d$), inside ($0\leqslant\xi\leqslant d$) and behind ($\xi<0$) the bunch are obtained in similar manner:

$\xi>d$;

$$\begin{aligned}\mathcal{E}^+(\xi) &= -i\widetilde{E}_0\frac{n_b}{n_0}(k_pd)\frac{\sqrt{a^4-1}}{a}\frac{\beta\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}+\widetilde{E}_0\frac{n_b}{n_0}\frac{\beta^2\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2}\frac{\sqrt{a^4-1}}{a}\times &(72)\\ &\times\left\{\left[2\alpha\delta\left(a\Delta-2\beta\gamma^2F\right)-\frac{i\alpha}{\alpha^2+\delta^2}\left(\left(\alpha^2+\delta^2\right)^2+2\beta\gamma^2Fa\Delta\left(3\delta^2-\alpha^2\right)\right)\right]\times\right.\\ &\times\left[\exp\left(-\frac{\delta}{a}k_p(\xi-d)\right)\cos\left(\frac{\alpha}{a}k_p(\xi-d)\right)-\exp\left(-\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)\right]+\\ &+\left[\left(a\Delta-2\beta\gamma^2F\right)\left(\alpha^2-\delta^2\right)+\frac{i\delta}{\alpha^2+\delta^2}\left(\left(\alpha^2+\delta^2\right)^2-2\beta\gamma^2Fa\Delta\left(3\alpha^2-\delta^2\right)\right)\right]\times\\ &\left.\times\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p(\xi-d)\right)\sin\left(\frac{\alpha}{a}k_p(\xi-d)\right)\right]\right\},\end{aligned}$$

$$\begin{aligned}\mathcal{B}^+(\xi) &= \widetilde{E}_0\frac{n_b}{n_0}(k_pd)\frac{\sqrt{a^4-1}}{a^2}\frac{\beta FF_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}+\widetilde{E}_0\frac{n_b}{n_0}\frac{\sqrt{a^4-1}}{a}\frac{\beta\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2}\times &(73)\\ &\times\left\{-\alpha\left[\frac{\left(\alpha^2+\delta^2\right)^2+2\beta^2\gamma^2\left(3\delta^2-\alpha^2\right)FF_0}{\alpha^2+\delta^2}+2i\beta\delta\left(F_0-2\gamma^2F\right)\right]\times\right.\\ &\times\left[\exp\left(-\frac{\delta}{a}k_p(\xi-d)\right)\cos\left(\frac{\alpha}{a}k_p(\xi-d)\right)-\exp\left(-\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)\right]+\end{aligned}$$

$$
\begin{aligned}
&+\left[\delta\frac{\left(\alpha^2+\delta^2\right)^2-2\beta^2\gamma^2\left(3\alpha^2-\delta^2\right)FF_0}{\alpha^2+\delta^2}+i\beta\left(\delta^2-\alpha^2\right)\left(F_0-2\gamma^2F\right)\right]\times \\
&\times\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p(\xi-d)\right)\sin\left(\frac{\alpha}{a}k_p(\xi-d)\right)\right]\Bigg\}.
\end{aligned}
$$

$0 \leqslant \xi \leqslant d$;

$$
\begin{aligned}
\mathcal{E}^+(\xi) \;=\;& i\widetilde{E}_0\frac{n_b}{n_0}\left(k_p(d-2\xi)\right)\frac{\sqrt{a^4-1}}{a}\frac{\beta\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}-\widetilde{E}_0\frac{n_b}{n_0}\frac{\beta^2\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2}\frac{\sqrt{a^4-1}}{a}\times \\
&\times\Bigg\{2\alpha\delta\left(a\Delta-2\beta\gamma^2F\right)\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\cos\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]+ \\
&+\left(a\Delta-2\beta\gamma^2F\right)\left(\delta^2-\alpha^2\right)\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\sin\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]- \\
&-\frac{i\alpha}{\alpha^2+\delta^2}\left[\left(\alpha^2+\delta^2\right)^2-2\beta\gamma^2Fa\Delta\left(\alpha^2-3\delta^2\right)\right]\times \\
&\times\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)+\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\cos\left(\frac{\alpha}{a}k_p(d-\xi)\right)-2\right]- \\
&-\frac{i\delta}{\alpha^2+\delta^2}\left[\left(\alpha^2+\delta^2\right)^2-2\beta\gamma^2Fa\Delta\left(3\alpha^2-\delta^2\right)\right]\times \\
&\times\left[\exp\left(-\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)+\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\sin\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]\Bigg\},
\end{aligned}
\tag{74}
$$

$$
\begin{aligned}
\mathcal{B}^+(\xi) \;=\;& -\widetilde{E}_0\frac{n_b}{n_0}\frac{\sqrt{a^4-1}}{a^2}\left(k_p(d-2\xi)\right)\frac{\beta FF_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}+\widetilde{E}_0\frac{n_b}{n_0}\frac{\sqrt{a^4-1}}{a}\frac{\beta\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2}\times \\
&\times\Bigg\{-\alpha\left[\frac{\left(\alpha^2+\delta^2\right)^2+2\beta^2\gamma^2\left(3\delta^2-\alpha^2\right)FF_0}{\alpha^2+\delta^2}+2i\beta\delta\left(F_0-2\gamma^2F\right)\right]\left[1-\exp\left(-\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)\right]+ \\
&+\alpha\left[\frac{\left(\alpha^2+\delta^2\right)^2+2\beta^2\gamma^2\left(3\delta^2-\alpha^2\right)FF_0}{\alpha^2+\delta^2}-2i\beta\delta\left(F_0-2\gamma^2F\right)\right]\left[1-\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\cos\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right] \\
&+\left[\delta\frac{\left(\alpha^2+\delta^2\right)^2-2\beta^2\gamma^2\left(3\alpha^2-\delta^2\right)FF_0}{\alpha^2+\delta^2}+i\beta\left(\delta^2-\alpha^2\right)\left(F_0-2\gamma^2F\right)\right]\exp\left(-\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)- \\
&-\left[\delta\frac{\left(\alpha^2+\delta^2\right)^2-2\beta^2\gamma^2\left(3\alpha^2-\delta^2\right)FF_0}{\alpha^2+\delta^2}-i\beta\left(\delta^2-\alpha^2\right)\left(F_0-2\gamma^2F\right)\right]\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\sin\left(\frac{\alpha}{a}k_p(d-\xi)\right)
\end{aligned}
\tag{7}
$$

$\xi < 0$;

$$
\begin{aligned}
\mathcal{E}^+(\xi) \;=\;& i\widetilde{E}_0\frac{n_b}{n_0}(k_pd)\frac{\sqrt{a^4-1}}{a}\frac{\beta\Delta F}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}}-\widetilde{E}_0\frac{n_b}{n_0}\frac{\beta^2\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2}\frac{\sqrt{a^4-1}}{a}\times \\
&\times\Bigg\{\alpha\left[2\delta\left(a\Delta-2\beta\gamma^2F\right)+i\frac{\left(\alpha^2+\delta^2\right)^2-2\beta\gamma^2Fa\Delta\left(\alpha^2-3\delta^2\right)}{\alpha^2+\delta^2}\right]\times \\
&\times\left[\exp\left(\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right)-\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\cos\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]+ \\
&+\left[\left(a\Delta-2\beta\gamma^2F\right)\left(\alpha^2-\delta^2\right)+i\delta\frac{2\beta\gamma^2Fa\Delta\left(3\alpha^2-\delta^2\right)-\left(\alpha^2+\delta^2\right)^2}{\alpha^2+\delta^2}\right]\times \\
&\times\left[\exp\left(\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right)+\exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\sin\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]\Bigg\},
\end{aligned}
\tag{76}
$$

$$
\begin{aligned}
\mathcal{B}^{+}(\xi) \;\; = \;\; & -\widetilde{E}_0 \frac{n_b}{n_0}\frac{\sqrt{a^4-1}}{a^2}(k_p d)\frac{\beta F F_0}{4F^2+\frac{a^4-1}{a^4}\gamma^{-2}} + \widetilde{E}_0 \frac{n_b}{n_0}\frac{\sqrt{a^4-1}}{a}\frac{\beta\gamma^2}{4\alpha\delta\left(\alpha^2+\delta^2\right)^2} \qquad (77)\\
& \times\left\{\alpha\left[\frac{\left(\alpha^2+\delta^2\right)^2+2\beta^2\gamma^2\left(3\delta^2-\alpha^2\right)FF_0}{\alpha^2+\delta^2} - 2i\beta\delta\left(F_0-2\gamma^2F\right)\right]\times\right.\\
& \times\left[\exp\left(\frac{\delta}{a}k_p\xi\right)\cos\left(\frac{\alpha}{a}k_p\xi\right) - \exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\cos\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right] -\\
& -\left[\delta\frac{\left(\alpha^2+\delta^2\right)^2-2\beta^2\gamma^2\left(3\alpha^2-\delta^2\right)FF_0}{\alpha^2+\delta^2} - i\beta\left(\delta^2-\alpha^2\right)\left(F_0-2\gamma^2F\right)\right]\times\\
& \left.\times\left[\exp\left(\frac{\delta}{a}k_p\xi\right)\sin\left(\frac{\alpha}{a}k_p\xi\right) + \exp\left(-\frac{\delta}{a}k_p(d-\xi)\right)\sin\left(\frac{\alpha}{a}k_p(d-\xi)\right)\right]\right\}.
\end{aligned}
$$

As it follows from the expression (56), in regions I and II the longitudinal field ahead of bunch ($\xi > d$) is missing. The transverse fields ahead of the bunch are independent of ξ (i.e., are not modulated), are circularly polarized and proportional to the function F and thickness of the bunch d. Note that F is proportional to the difference between the group velocity of induced transverse fields ($v_g = k_0c^2/\omega_0$) and the bunch velocity. If the group velocity of the transverse wave coincides with the bunch velocity ($F = 0$), then the transverse fields ahead of the bunch are absent. Besides, the magnetic field ahead of the bunch is proportional to F_0 and is absent in the long-wave (quasi-stationary) limit ($k_0 = 0$ or $F_0 = 0$).

The transverse fields ahead of the bunch arise on account of the fact that the CPEM wave has a phase velocity ($v_\varphi = \omega_0/k_0 > c$) exceeding the bunch velocity at any values of parameters of plasma and of the CPEM wave. Hence, some part of perturbations caused in plasma by the CPEM wave and bunch may have phase velocity higher than the bunch velocity and be even ahead of it.

Inside and behind of bunch for values of parameters in regions I, II, III, as well as ahead of it in the region III (see the Eqs. (59), (61), (63), (64), (66), (67), (69), (70), (72)-(77)) the transverse fields are modulated and are circularly polarized. Though the polarization vector of the transverse wave describes a circle, its radius depends on the distance to the bunch ξ. Indeed, as it follows from Eqs. (29)-(31)

$$
E_x'^2 + E_y'^2 = E_r^2(\xi) + E_i^2(\xi) = E_{\max}^2(\xi), \tag{78}
$$

$$
B_x'^2 + B_y'^2 = B_r^2(\xi) + B_i^2(\xi) = B_{\max}^2(\xi). \tag{79}
$$

The amplitudes of electric ($E_{\max}(\xi)$) and magnetic ($B_{\max}(\xi)$) fields are generally dependent on ξ. The modulated transverse waves in plasma arise in consequence of the excitation of two type waves, having frequencies $\omega_L\lambda_\pm$ and wave vectors $(\omega_L/u)\lambda_\pm$. Owing to the interaction of the pump wave with these induced waves, there arise waves with combined frequencies $\omega_0 - \omega_L\lambda_\pm$, $\omega_0 + \omega_L\lambda_\pm$ and combined wave vectors $k_0 - (\omega_L/u)\lambda_\pm$, $k_0 + (\omega_L/u)\lambda_\pm$, the interference between which gives a modulated wave.

Here $E_r(\xi)$ or $E_i(\xi)$ serve as a carrier wave if $\omega_0 > \omega_L\lambda_\pm$ (in the dimensionless form $a\Delta > \lambda_\pm$) and $k_0 > (\omega_L/u)\lambda_\pm$ (in the dimensionless form $\beta\sqrt{a^2\Delta^2-1} > \lambda_\pm$). Otherwise in expressions (29) and (30) the part of the carrier wave will play the functions $\cos\zeta$ and $\sin\zeta$.

Below we shall analyze the obtained expressions in practically important case of the following values of parameters: $n_0 \lesssim 10^{17}\,\mathtt{cm}^{-3}$, ($\omega_p \lesssim 2\times10^{13}\,\mathrm{sec}^{-1}$), $W_L \simeq 10^{18}\mathrm{W/cm}^2 \div 10^{20}\mathrm{W/cm}^2$ ($W_L = cE_0^2/4\pi$ is the intensity of CPEM wave), $\omega_0 \simeq 10^{15}\,\mathrm{sec}^{-1}$, $\gamma \simeq 10\div10^3$. At the variation of CPEM wave intensity in the range from

$10^{18}\mathrm{W/cm^2}$ to $10^{20}\mathrm{W/cm^2}$ the parameter a is changed in the interval $a \simeq 1.02 \div 2$. For the parameter Δ we have: $\Delta \gtrsim 50$.

In the region I $\gamma \lesssim \Delta$ or $\gamma \gtrsim \Delta$ (the Eq. (40)). It is comparatively easy to analyze the Eqs. (56)-(64) in two particular cases of $a\Delta \gg \gamma \gg 1$ and $1 \ll a\Delta \ll \gamma$ respectively. In the first case we obtain from Eqs. (34) and (35):

$$4\gamma^4 F^2 \simeq a^2\Delta^2 \left(1 - \frac{2\gamma^2}{a^2\Delta^2}\right) \gg \gamma^2 \gg 1, \tag{80}$$

$$\lambda_-^2 \simeq 1, \quad \lambda_+^2 \simeq 4\gamma^4 F^2 \simeq a^2\Delta^2 \gg \lambda_-^2. \tag{81}$$

The transverse electric and magnetic fields ahead of the bunch are on the order of magnitude $|\mathcal{E}^+| \simeq |\mathcal{B}^+| \simeq \widetilde{E}_0(n_b/n_0)(k_p d)\gamma^2/2$ when $a \gtrsim 1$. Inside and behind the bunch the amplitude of induced waves with frequency $\omega_L\lambda_+$ is much less than the one for waves with frequency $\omega_L\lambda_-$. By the order of magnitude the amplitude of oscillations with frequency $\omega_L\lambda_-$ in the longitudinal waves (Eqs. (59) and (62) is $\widetilde{E}_0(n_b/n_0)a$, i.e., is $\gamma^2(k_p d)/2a$ times as less as the amplitude of transverse waves ahead of the bunch. For $0 \leqslant \xi \leqslant d$ and $\xi < 0$ the highest contributions to expressions (60), (61), (63), (64) are made by the first and third terms, the amplitude of the third term being approximately equal to $\widetilde{E}_0(n_b/n_0)\gamma^2 a$ (i.e., γ^2 times as much as the amplitude of longitudinal waves). Hence, in the region I $E_i(\xi) \gg E_r(\xi) \simeq 0$, $B_r(\xi) \gg B_i(\xi) \simeq 0$ and $E_x^{'} \cong -E_i(\xi)\sin\zeta$, $E_y^{'} \cong E_i(\xi)\cos\zeta$, $B_x^{'} \cong B_r(\xi)\cos\zeta$, $B_y^{'} \cong B_i(\xi)\sin\zeta$. Besides, the oscillating terms in Eqs. (59)-(64) describe the waves, the length of which increases with the intensity of CPEM wave. Third, when the condition $\pi d < a\lambda_p$ is met (where $\lambda_p = 2\pi/k_p$ is the wavelength of excited longitudinal waves in the absence of the CPEM wave), the oscillating terms in expressions (60), (61), (63), (64) exceed the first terms for narrow bunches. In case of wide bunches ($\pi d > a\lambda_p$) the first term in Eqs. (63) and (64) exceeds the third term. Inside the wide bunch the analogous conclusion is valid close to the bunch boundaries when $\pi\,|d/2 - \xi| > a\lambda_p/2$. Inside the wide bunch, in the vicinity of its center, $\pi\,|d/2 - \xi| < a\lambda_p/2$, the main contribution to Eqs. (60) and (61) is made by the third terms.

Behind the bunch, the amplitude of longitudinal and transverse waves is proportional to $2\sin(\pi d/a\lambda_p)$ and is maximum at $d = (n - 1/2)a\lambda_p$, where $n = 1, 2, \ldots$. When the condition $d = (a\lambda_p)n$ is fulfilled, the waves behind the bunch are not excited.

In Figs. 4-10 are shown the results of numerical calculations made for induced fields using Eqs. (56)-(64). The figures illustrate all the features discussed earlier. Thus, in the presence of a CPEM wave in the plasma, the one-dimensional bunch excites the waves with the wavelengths increasing as the intensity of CPEM wave. The dependence of longitudinal wave amplitude on the energy of bunch (on the relativistic factor) is weak and grows with the intensity of CPEM wave, whereas the amplitude of the transverse wave is γ^2 times as large as that of longitudinal wave. One can assert that at $\gamma \gg 1$ the induced wave is almost a transverse one.

Now consider another limiting case with $\gamma \gg a\Delta \gg 1$. Here instead of Eqs. (80) and (81) we find

$$4\gamma^4 F^2 \simeq \gamma^2 \left(\frac{\gamma}{a\Delta}\right)^2 \gg 1, \tag{82}$$

$$\lambda_-^2 \simeq 1, \quad \lambda_+^2 \simeq \gamma^2 \left(\frac{\gamma}{a\Delta}\right)^2 \gg \lambda_-^2. \tag{83}$$

When $a \gtrsim 1$, the transverse fields ahead of the bunch are by the order of magnitude: $|\mathcal{E}^+| \simeq |\mathcal{B}^+| \simeq \widetilde{E}_0(n_b/n_0)(k_p d)a^2\Delta^2/2$. Inside and behind the bunch the

amplitude of waves with frequency $\omega_L\lambda_+$ is again much less than the amplitude of waves with frequency $\omega_L\lambda_-$. The amplitude of oscillations with frequency $\omega_L\lambda_-$ in the longitudinal waves (Eqs. (59) and (62)) does not change and is nearly equal to $\widetilde{E}_0(n_b/n_0)a$. In this case the magnitude of the longitudinal wave is by $(k_pd)a\Delta^2/2$ times less than the transverse wave amplitude ahead of the bunch. Inside and behind the bunch the primary role in the transverse waves is played again by the first and third terms. The amplitude of the third terms is $\widetilde{E}_0(n_b/n_0)a(a\Delta)^2$ (i.e., by $(a\Delta)^2$ times larger than the amplitude of longitudinal waves). Note that in the limit of very large values of γ ($\gamma \gg a\Delta$) the transverse fields are independent of it, but the dependence on intensity of CPEM wave becomes more salient. All the remaining features mentioned above for the case of $\gamma \ll a\Delta$ are still valid here.

As we saw in Sec. III, $\gamma \sim a\Delta$ in the second region of values of a, Δ, γ. In this case the function F in Eqs. (56)-(58), (65)-(70) takes on the values $F_\pm$ (see the expression (38)), the region II being determined by expressions (36) for $\gamma \gg 1$ and an arbitrary value of parameter a, and by expressions (37) for $\gamma \gg 1$ and $1 < a < a_0(\gamma)$ (where $a_0(\gamma)$ is determined by the expression (39)). The values $F = \pm F_+$ correspond to Eq. (36), and the values $F = \pm F_-$ correspond to Eq. (37). Respectively, in the expression (45) one is to take the sign "+" if $F = \pm F_+$, and the sign "−" if $F = \pm F_-$. From expression (45) for values $\gamma \gg 1$, $\Delta \gg 1$ we have $\lambda_0 \simeq \gamma^{1/2} \gg 1$ if $a \gtrsim 1 + 1/4\gamma^2$, and $\lambda_0 \lesssim 1$ if $1 < a < a_0(\gamma) \simeq 1 + 1/16\gamma^2$. In the first case the wavelength of the waves excited inside and behind the bunch is decreased as γ increases and is much less than the wavelength in the second case, when that is on the order of $a\lambda_p$.

As it follows from Eqs. (65)-(70), at the distance from the bunch the amplitudes of waves to be excited increase proportional to the distance to bunch and beginning from some value $\xi = \xi_0$ the fields may be of the same order of magnitude as the amplitude of CPEM wave. Surely, in case of $\xi > \xi_0$ the Eqs. (68)-(70) are not adequate for the description of induced fields and for their treatment one is to avail of the nonlinear approach.

The obtained results for region II were further analyzed numerically. In Figs. 11-14 the coordinate dependence of induced longitudinal and transverse fields for the following values of CPEM wave, plasma and bunch parameters is shown: $n_b = 10^{14}\mathrm{cm}^{-3}$ (Fig. 11), $n_b = 10^{13}\mathrm{cm}^{-3}$ (Figs. 12-14), $n_0 = 10^{17}\mathrm{cm}^{-3}$, $\omega_0 = 3.77 \times 10^{15}\mathrm{sec}^{-1}$, $\gamma = 210$, $d = 2k_p^{-1}$, $k_p^{-1} = 1.7 \times 10^{-3}\mathrm{cm}$. The strength of CPEM wave field is varied from the value $E_0 = 8.5 \times 10^8\mathrm{V/cm}$ to $E_0 = 1.85 \times 10^{10}\mathrm{V/cm}$. The ξ-dependence of $B_{\max}$ in Figs. 11-14 is not shown, since the difference of $B_{\max}$ from $E_{\max}(\xi)$ is insignificant as it follows from Eqs. (66), (67) and (69), (70) for $a\Delta \gg 1$, $\gamma \gg 1$. From Fig. 12 one can estimate the distance ξ_0, at which the amplitude of transverse waves is of the order of CPEM wave amplitude and the linear approach is of no sense any more. From Fig. 12 one gets the following estimate: $\xi_0 \simeq 60k_p^{-1}$. So, the linear treatment is valid at distances of several wavelengths $(2\pi/k_p)$.

In the region III the fields are described by Eqs. (71)-(77). At large distances from the bunch $|\xi| \gtrsim a(\delta k_p)^{-1}$, $|\xi - d| \gtrsim a(\delta k_p)^{-1}$, the longitudinal waves are attenuated, and transverse waves are described by the first terms in Eqs. (72), (73) and (76), (77) that are independent of ξ (i.e., are not modulated). The ξ-dependence of transverse and longitudinal waves is given in Figs. 15-21. It follows from these figures that the longitudinal field and amplitudes E_r, E_i, B_r are antisymmetric with respect to the bunch center ($\xi = d/2$ plane), and the amplitude B_i is symmetric with respect to it. Besides, at large distances the phase of transverse waves ahead of the bunch differs from the phase of transverse wave behind the bunch by π, and the transverse oscillations behind and ahead of the bunch are in the counterphase.

5 Summary

The purpose of this work was to investigate the excitation of EM wake waves in electron plasma by one-dimensional bunch of charged particles in the presence of intense CPEM wave. The interaction of the pump wave with a plasma was described by means of Maxwell equations and relativistic nonlinear hydrodynamic equations of plasma. Then, we have considered small perturbations of plasma due to the presence of one-dimensional bunch. A general expressions obtained for EM wake field components was analyzed in three particular ranges of parameters of the pump wave, bunch and plasma. In the first range of parameters the amplitude of transverse components of induced waves is shown to grow as the bunch energy and after some definite value of the relativistic factor of the bunch to be almost independent of the energy and increase proportional to the intensity and frequency of the CPEM wave. The dependence of the longitudinal component of induced waves on the relativistic factor of the bunch is weak. Its amplitude and wavelength grow as the intensity of CPEM wave. In the second range of parameters the amplitude of the wave excited by the bunch is a linear function of the distance to the bunch. In the third range of parameters the longitudinal component of induced fields is localized near the bunch boundaries and exponentially decreases with increasing distance from these boundaries. The amplitudes of transverse components of induced waves reach a constant value with the distance from the bunch boundaries. In general, the transverse waves in all three mentioned ranges of values of the CPEM wave, plasma and bunch parameters are modulated and are circularly polarized with the same type (right-hand or left-hand) of polarization as the pump wave.

In conclusion it is worthwhile to make two remarks. First, although at the consideration of EM wake wave generation we took the external CPEM wave to be strong, it was assumed monochromatic. The intensity of present-day sources of monochromatic EM waves does not exceed 10^{16}W/cm^2 and the values of amplitudes used for numerical estimates and calculations are much higher than this value. In the nearest future we plan to study the excitation of EM wake waves by one-dimensional bunch in the presence of high power laser pulses with the intensity ~10^{20}W/cm^2. The results of this study will be presented in a forthcoming work. Second, it follows from Eqs. (10)-(14) that in the plasma there arise parametrically coupled linear waves. In case of high bunch densities ($n_b \gtrsim n_0$) the linear treatment is of nosense any more and it is necessary to examine explicit system of nonlinear Eqs. (1)-(5). This issue will also be studied in future work.

ACKNOWLEDGMENT

The work has been supported by the International Science and Technology Center under Project No. A-013.

6 Appendix

In Section II a set of inhomogeneous differential equations with constant coefficients for induced variables $\mathcal{E}^{\pm}$, $B^{\pm}$, $\mathcal{V}^{\pm}$, $E_z^{'}$ and $n^{'}$ (see Eqs. (16)-(20)) has been found. The solution of this set of equations was obtained with the help of Green's function. Below we shall briefly discuss another method for solution of the set of Eqs. (16)-(20), i.e., we shall obtain a differential equation for one of the induced quantities, i.e., for the density of induced charge $n^{'}$. The equations for other variables will be obtained in an analogous way.

Now consider the Eq. (20) and find the differential operator the action of which on $\mathcal{B}^{+}+\mathcal{B}^{-}$ and $\mathcal{V}^{+}+\mathcal{V}^{-}$ give a function of $n^{'}$. Let us introduce the operators

$$\widehat{Q}^{\pm}=\frac{1}{\gamma^2}\frac{\partial^2}{\partial\xi^2}\mp 2i\left(k_0-\beta\frac{\omega_0}{c}\right)\frac{\partial}{\partial\xi}+\frac{\omega_L^2}{c^2}. \tag{A.1}$$

After successive action of operators $\widehat{Q}^{-}$ and $\widehat{Q}^{+}$ on both the sides of the Eq. (18) we obtain the equation that comprises only the functions $\mathcal{V}^{\pm}(\xi)$ and $n^{'}(\xi)$:

$$\widehat{Q}^{+}\widehat{Q}^{-}\mathcal{V}^{\pm}(\xi)=\frac{\omega_L^2}{c^2}\left[\left(1-\frac{\beta_e^2}{2}\right)\widehat{Q}^{\mp}\left(\mathcal{V}^{\pm}(\xi)\pm iv_e\frac{n^{'}(\xi)}{n_0}\right)+\frac{\beta_e^2}{2}\widehat{Q}^{\pm}\left(\mathcal{V}^{\mp}(\xi)\mp iv_e\frac{n^{'}(\xi)}{n_0}\right)\right]. \tag{A.2}$$

Now, introduce another operator $\widehat{S}$, that is symmetric with respect to operators $\widehat{Q}^{-}$ and $\widehat{Q}^{+}$, and indeed:

$$\widehat{S}=\widehat{Q}^{+}\widehat{Q}^{-}-\frac{\omega_L^2}{c^2}\left(1-\frac{\beta_e^2}{2}\right)\left(\widehat{Q}^{-}+\widehat{Q}^{+}\right)+\frac{\omega_L^4}{c^4}\left(1-\beta_e^2\right). \tag{A.3}$$

After some transformations one can write the Eq. (A.2) with the help of operator $\widehat{S}$

$$\widehat{S}\mathcal{V}^{\pm}(\xi)=\pm iv_e\frac{\omega_L^2}{c^2}\left[\left(1-\frac{\beta_e^2}{2}\right)\widehat{Q}^{\mp}-\frac{\beta_e^2}{2}\widehat{Q}^{\pm}-\frac{\omega_L^2}{c^2}\left(1-\beta_e^2\right)\right]\frac{n^{'}(\xi)}{n_0}. \tag{A.4}$$

From Eqs. (A.4) and (17) the required equation follows for variables $\mathcal{V}^{-}+\mathcal{V}^{+}$ and $\mathcal{B}^{-}+\mathcal{B}^{+}$:

$$\widehat{S}\left[\mathcal{V}^{-}(\xi)+\mathcal{V}^{+}(\xi)\right]=-4v_e\frac{\omega_L^2}{c^2}\left(k_0-\beta\frac{\omega_0}{c}\right)\frac{\partial}{\partial\xi}\frac{n^{'}(\xi)}{n_0}, \tag{A.5}$$

$$\widehat{S}\left[\mathcal{B}^{-}(\xi)+\mathcal{B}^{+}(\xi)\right]=-4\pi en_0\beta_e\left\{\left(\widehat{Q}^{-}+\widehat{Q}^{+}\right)\frac{\partial}{\partial\xi}+ik_0\left(\widehat{Q}^{+}-\widehat{Q}^{-}\right)-2\frac{\omega_L^2}{c^2}\frac{\partial}{\partial\xi}\right\}\frac{n^{'}(\xi)}{n_0}. \tag{A.6}$$

Now, acting by the operator $\widehat{S}$ on both the sides of the Eq. (20) and using the relations (A.5), (A.6), we finally arrive at the equation that contains only the variable $n^{'}$

$$\left(\frac{\partial^4}{\partial\xi^4}+A\frac{\partial^2}{\partial\xi^2}+B\right)\frac{n^{'}(\xi)}{n_0}=-\frac{\omega_L^2}{u^2}\left(\frac{\partial^2}{\partial\xi^2}+C\right)\frac{N_b(\xi)}{n_0}, \tag{A.7}$$

where

$$A = \frac{\omega_L^2}{u^2}\left(1-\beta_e^2\right) + 4\gamma^4\left(k_0 - \beta\frac{\omega_0}{c}\right)^2, \tag{A.8}$$

$$B = \frac{\omega_L^2}{u^2}\gamma^2\left[\frac{\beta_e^2\omega_L^2}{c^2} + 4\gamma^2\left(k_0 - \beta\frac{\omega_0}{c}\right)^2\right], \tag{A.9}$$

$$C = \gamma^2\left[\frac{\beta_e^2\omega_L^2}{c^2} + 4\gamma^2\left(k_0 - \beta\frac{\omega_0}{c}\right)^2\right]. \tag{A.10}$$

One can obtain the equations for variables $\mathcal{E}^{\pm}$, $\mathcal{B}^{\pm}$, $\mathcal{V}^{\pm}$, $E_z^{'}$ in an analogous way.

Prior to the consideration of Eq. (A.7) note first of all that in the absence of CPEM wave ($\beta_e = 0$) one has the following relation from Eqs. (A.7)-(A.10):

$$\left(\frac{\partial^4}{\partial\xi^4} + A\frac{\partial^2}{\partial\xi^2} + B\right) = \left(\frac{\partial^2}{\partial\xi^2} + C\right)\left(\frac{\partial^2}{\partial\xi^2} + \frac{\omega_p^2}{u^2}\right) \tag{A.11}$$

and Eq. (A.7) passes into the well known equation for the induced charge obtained in Refs. [24, 25]

$$\left(\frac{\partial^2}{\partial\xi^2} + \frac{\omega_p^2}{u^2}\right) n^{'}(\xi) = -\frac{\omega_p^2}{u^2}N_b(\xi). \tag{A.12}$$

In the limit of strong CPEM wave ($\beta_e \simeq 1$) we have from expressions (A.7)-(A.10)

$$A \simeq C \simeq 4\frac{\omega_0^2}{c^2}(1+\beta)^{-2}, \ \ B \simeq 0, \tag{A.13}$$

$$\frac{\partial^2}{\partial\xi^2}\left(\frac{\partial^2}{\partial\xi^2} + q_0^2\right) n^{'}(\xi) = 0, \tag{A.14}$$

where $q_0 = \sqrt{A} = 2(\omega_0/c)(1+\beta)^{-1}$. It follows from expression (A.14) that in this case the equation for $n^{'}$ will be a homogeneous differential equation. Analogous equations are obtained for the rest of the induced variables. So, in the presence of high intensity CPEM wave the bunch will not altogether perturb the state of homogeneous plasma established by the external wave and all induced variables will be zeros.

For an external wave of arbitrary intensity a non-homogeneous equation of the fourth degree is obtained for $n^{'}(\xi)$ (Eq. (A.7)), the coefficients of which depend on the intensity of CPEM wave. The characteristic equation corresponding to Eq. (A.7) establishes in general case a dispersion law for two types of induced waves that are parametrically coupled due to the presence of the external wave (it is easy to see that it coincides with Eq. (33)). In case of $A^2 > 4B$ Eq. (A.7) describes the oscillations of induced charge density with frequencies of $\omega_L\lambda_-$ and $\omega_L\lambda_+$ respectively. This type of solution corresponds to the first range of parameter values that was studied above. In case of $A^2 = 4B$ the characteristic equation has multiple solutions and respectively the solutions of Eq. (A.7) describe the waves, the amplitude of which increases as ξ (range II). At $A^2 < 4B$ the solutions of the characteristic equation are complex and the waves excited by the electron bunch exponentially decrease with the distance from the bunch (region III).

To obtain a single-valued solution of the problem one is to supplement Eq. (A.7) with boundary conditions. As such in case of an arbitrary, smoothly changing density profile of the bunch $N_b(\xi)$ when $\xi \to \pm\infty$, one can use the equality of function $n^{'}(\xi)$ and of its first three derivatives at $\xi \to +\infty$ (or $\xi \to -\infty$) to zero. A distinguishing feature of the one-dimensional bunch with sharp boundaries (the profile of this kind for $N_b(\xi)$ we have used in calculations of induced EM fields) is

the fact that the even derivatives of the function $n^{'}(\xi)$ are jump functions at the boundaries of the bunch. Indeed, $n^{'}(\xi)$ is continuous at the boundaries of the bunch (for $\xi = 0$ and $\xi = d$). As it follows from Eq. (20), $\partial n^{'}/\partial\xi$ is also continuous on the boundaries of the bunch. We shall obtain by integrating Eq. (A.7) over the variable ξ in an infinitesimal range in the vicinity of points $\xi = 0$ or $\xi = d$, that $\partial^3 n^{'}/\partial\xi^3$ is also continuous on the boundaries of the bunch. Now consider Eq. (A.7) in the points $\xi = \xi_0 - 0$ and $\xi = \xi_0 + 0$ (where $\xi_0 = 0$ or $\xi_0 = d$) respectively. The subtraction of the obtained relations gives the following condition:

$$\left(\frac{\partial^4 n^{'}}{\partial\xi^4} + A\frac{\partial^2 n^{'}}{\partial\xi^2}\right)_{\xi_0-0}^{\xi_0+0} = -\frac{\omega_L^2}{u^2}C\sigma n_b, \tag{A.15}$$

where $\sigma = +1$ for $\xi_0 = 0$ and $\sigma = -1$ for $\xi_0 = d$. Besides the condition (A.15) and the conditions of continuity of functions of $n^{'}$, $\partial n^{'}/\partial\xi$ and $\partial^3 n^{'}/\partial\xi^3$ at the boundaries of the bunch, there is another boundary condition on the induced variables describing the longitudinal waves in plasma (i.e., on $n^{'}(\xi)$, $v_z^{'}(\xi)$ and $E_z^{'}(\xi)$). According to this condition, owing to the equality of phase velocities of longitudinal waves induced in the plasma to the velocity of bunch ahead of these, the functions $n^{'}(\xi)$ (and, therefore, $v_z^{'}(\xi)$) and $E_z^{'}(\xi)$ turn zero. Thus, all the mentioned boundary conditions for a bunch with sharp boundaries together with Eq. (A.7) specify the unique solution of Eq. (A.7). The equations and boundary conditions for transverse induced variables ($\mathcal{E}^{\pm}$, $\mathcal{B}^{\pm}$, $\mathcal{V}^{\pm}$) the solutions for which coincide with expressions (22)-(25), may be obtained in an analogous way.

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FIGURE CAPTIONS

FIG. 1. The line $\Delta = \Delta(a)$, where the expression under the square root in Eq. (34) turns zero. The region III is contained within the lines, the region I is outside the lines, and the curve $\Delta = \Delta(a)$ is the region II. The plot is made for $\gamma = 1.5$.

FIG. 2. The line $\Delta = \Delta(a)$ for $\gamma = 20$.

FIG. 3. The line $\Delta = \Delta(a)$ for $\gamma = 100$.

FIG. 4. The ξ-dependence of the induced longitudinal electric field in the region I of the values of parameters. The field is measured in the units of 10^5V/cm, ξ is measured in units of $k_p^{-1} = 1.7\times10^{-3}$cm ($z_0 = k_p\xi$). The curves were calculated for the following values of parameters: $n_0 = 10^{17}\text{cm}^{-3}$, $n_b = 10^{14}\text{cm}^{-3}$, $\omega_0 = 3.77 \times 10^{15}\,\text{sec}^{-1}$, $\gamma = 50$, $k_p d = 20$. The dotted line corresponds to the absence of CPEM wave ($E_0 = 0$), the dashed line corresponds to the value $E_0 = 1.3 \times 10^{11}$V/cm, the solid line corresponds to the value $E_0 = 2.5 \times 10^{11}$V/cm.

FIG. 5. The ξ-dependence of maximum value of induced transverse electric field $\left(E_{\max} = \sqrt{E_r^2(\xi) + E_i^2(\xi)}\right)$in the region I. $E_{\max}$ is measured in units of 10^9V/cm, ξ in units of 1.7×10^{-3}cm. The thickness of the bunch is $10k_p^{-1}$. The dotted line corresponds to $E_0 = 6.7 \times 10^{10}$V/cm, the dashed line to $E_0 = 8.8 \times 10^{10}$V/cm, the solid line to $E_0 = 1.09 \times 10^{11}$V/cm. The remaining parameters coincide with parameters given in Fig. 4.

FIG. 6. The ξ-dependence of field $E_r(\xi)$ (in units of 10^5V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters and notations are the same as in Fig. 5.

FIG. 7. The ξ-dependence of field $E_i(\xi)$ (in units of 10^9V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters and notations are the same as in Fig. 5.

FIG. 8. The ξ-dependence of the maximum value of induced magnetic field $\left(B_{\max} = \sqrt{B_r^2(\xi) + B_i^2(\xi)}\right)$ (in units of 10^9V/cm, and in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters and notations are the same as in Fig. 5.

FIG. 9. The ξ-dependence of magnetic field $B_r(\xi)$ (in units of 10^9V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters and notations are the same as in Fig. 5.

FIG. 10. The ξ-dependence of $B_i(\xi)$ (in units of 10^5V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters and notations are the same as in Fig. 5.

FIG. 11. The ξ-dependence of induced longitudinal electric field in the region II (in units of 10^6V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The curves were obtained for values of parameters: $n_b = 10^{14}\text{cm}^{-3}$, $n_0 = 10^{17}\text{cm}^{-3}$, $\omega_0 = 3.77 \times 10^{15}\,\text{sec}^{-1}$, $\gamma = 210$, $k_p d = 2$. The dotted line corresponds to the value $E_0 = 8.5\times10^8$V/cm, the solid line corresponds to the value $E_0 = 1.85\times10^{10}$V/cm.

FIG. 12. The ξ-dependence of $E_{\max}$ in the region II (in units of 10^9V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm) for the values of parameters: $n_b = 10^{13}\text{cm}^{-3}$, $E_0 = 1.85 \times 10^{10}$V/cm. The remaining parameters are the same as in Fig. 11.

FIG. 13. The ξ-dependence of fields $E_r(\xi)$, $B_r(\xi)$ in the region II (in units of 10^9V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm) for the values of parameters: $n_b = 10^{13}\text{cm}^{-3}$, $k_p d = 2$. The values of remaining parameters are the same as in Fig. 11. The dotted line corresponds to the value of $E_r(\xi)$, the solid line corresponds to $B_r(\xi)$.

FIG. 14. The ξ-dependence of fields $E_i(\xi)$, $B_i(\xi)$ in the region II (in units of 10^9V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm). The parameters are the same as in Fig. 13. The dotted line corresponds to $E_i(\xi)$, the solid line corresponds to $B_i(\xi)$.

FIG. 15. The ξ-dependence of the induced longitudinal field in the region III (in units of 10^2V/cm, and ξ in units of $k_p^{-1} = 1.7\times10^{-3}$cm). The parameters are: $n_b = 10^{14}\text{cm}^{-3}$, $n_0 = 10^{17}\text{cm}^{-3}$, $\omega_0 = 3.77 \times 10^{15}\,\text{sec}^{-1}$, $\gamma = 210$, $k_p d = 5$. The dotted

line corresponds to the value $E_0 = 4.09 \times 10^9$V/cm, the dashed line corresponds to $E_0 = 1.16 \times 10^{10}$V/cm, the solid line corresponds to $E_0 = 1.6 \times 10^{10}$V/cm.

FIG. 16. The ξ-dependence of $E_{\max}$ in the region III (in units of 10^8V/cm, and ξ in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm) for $n_b = 10^{13}$cm^{-3}. The remaining parameters and notations are the same as given in Fig. 15.

FIG. 17. The ξ-dependence of the field $E_r(\xi)$ in region III (in units of 10^8V/cm, and ξ in units of $k_p^{-1} = 1.7\times10^{-3}$cm) for $n_b = 10^{13}$cm^{-3}. The remaining parameters and notations are the same as given in Fig. 15.

FIG. 18. The ξ-dependence of the field $E_i(\xi)$ in region III (in units of 10^8V/cm, and in units of $k_p^{-1} = 1.7 \times 10^{-3}$cm) for $n_b = 10^{13}$cm^{-3}. The remaining parameters and notations are the same as given in Fig. 15.

FIG. 19. The field $B_{\max}(\xi)$ in the region III. The values of parameters, measurement units and notations are the same as in Fig. 16.

FIG. 20. The field $B_r(\xi)$ in the region III. The values of parameters, measurement units and notations are the same as in Fig. 16.

FIG. 21. The field $B_i(\xi)$ in the region III. The values of parameters, measurement units and notations are the same as in Fig. 16.

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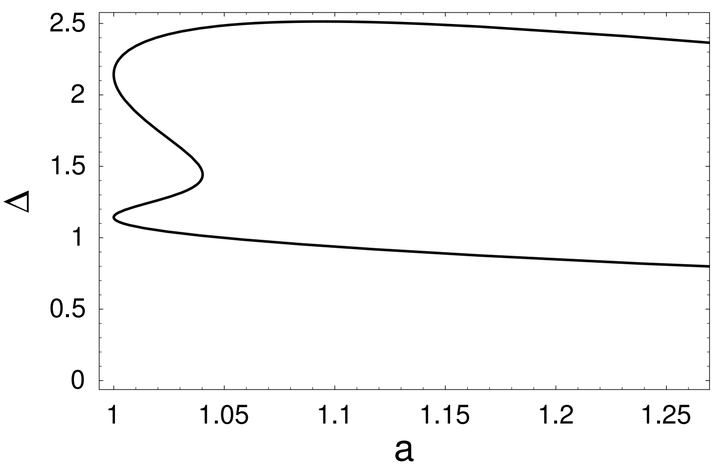

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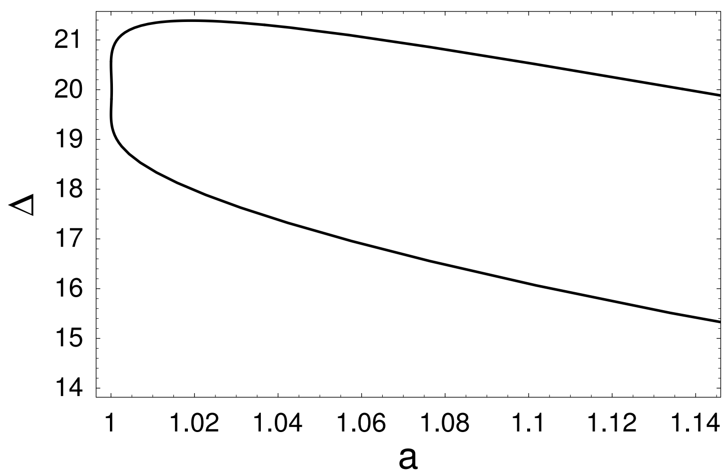

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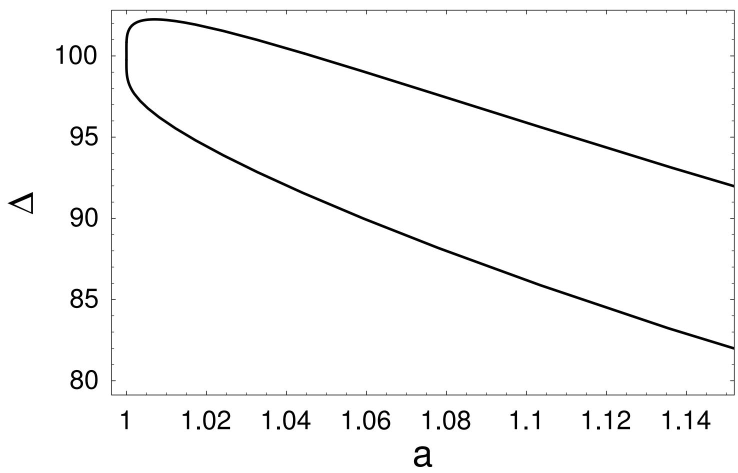

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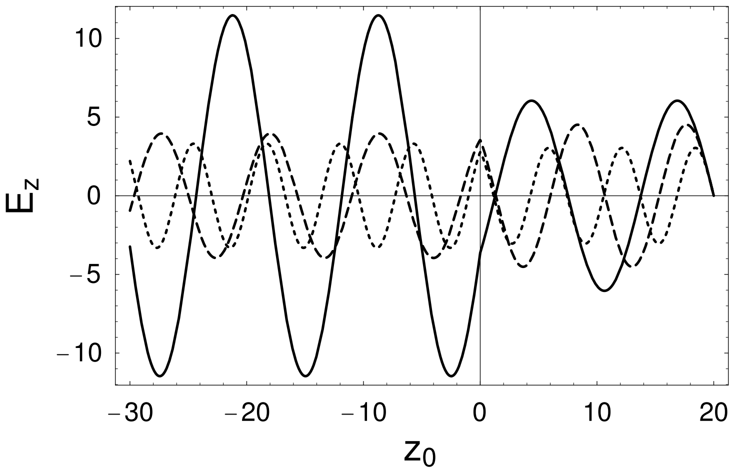

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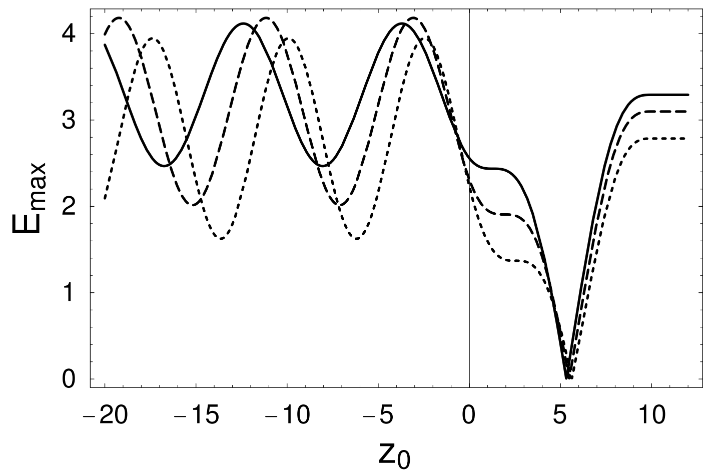

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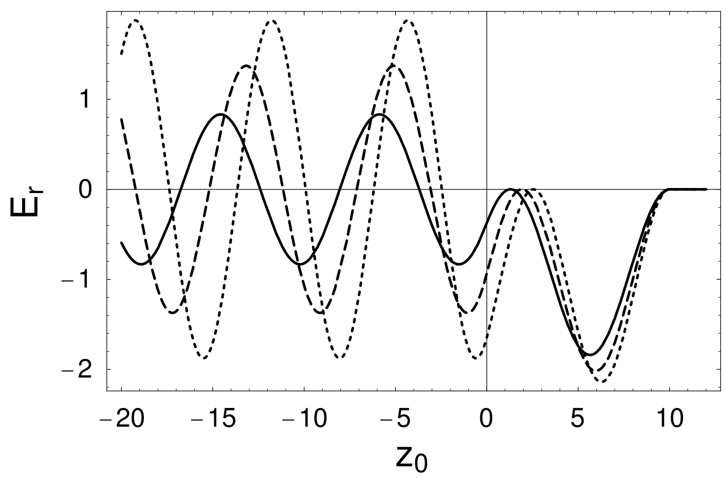
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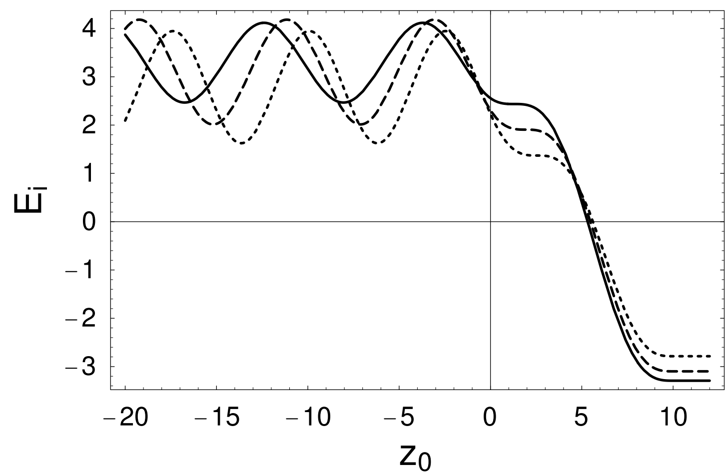

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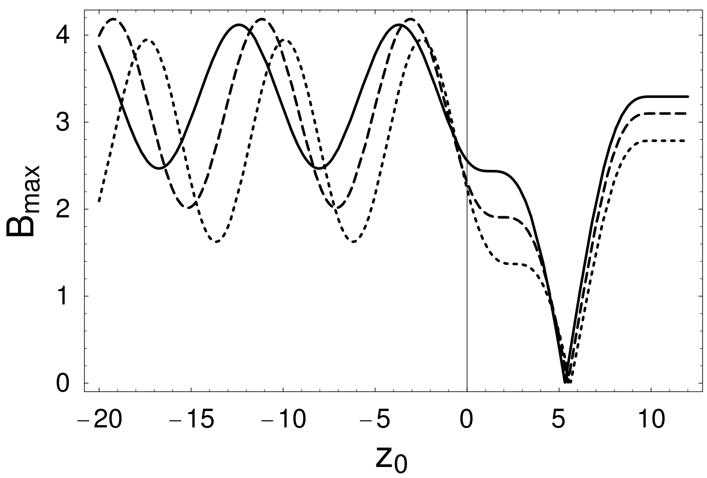

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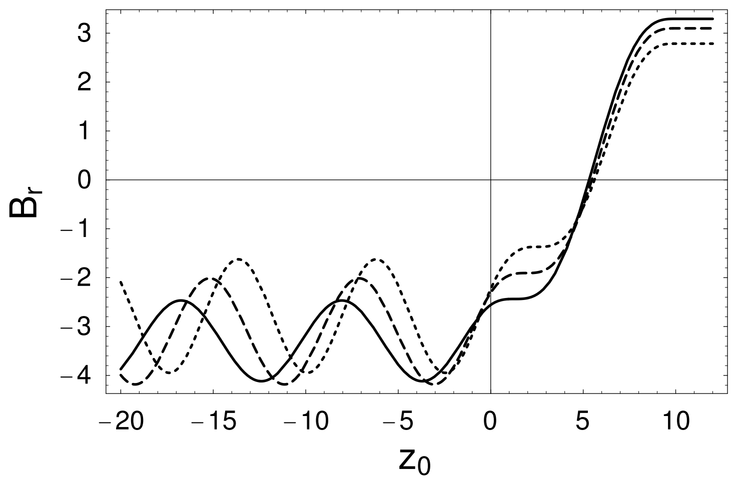

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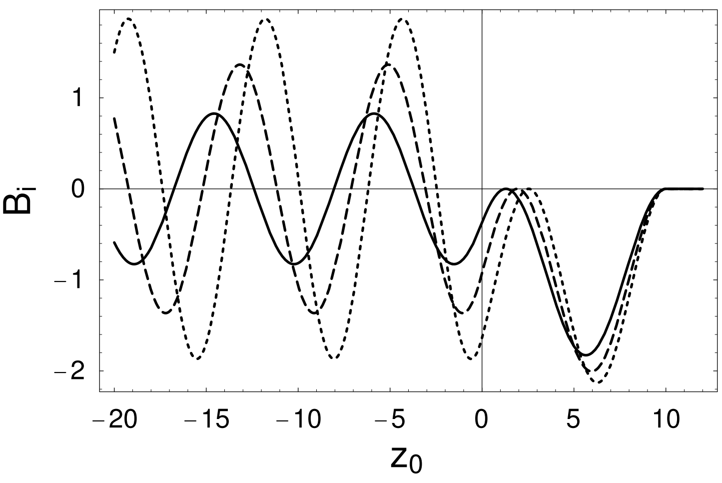

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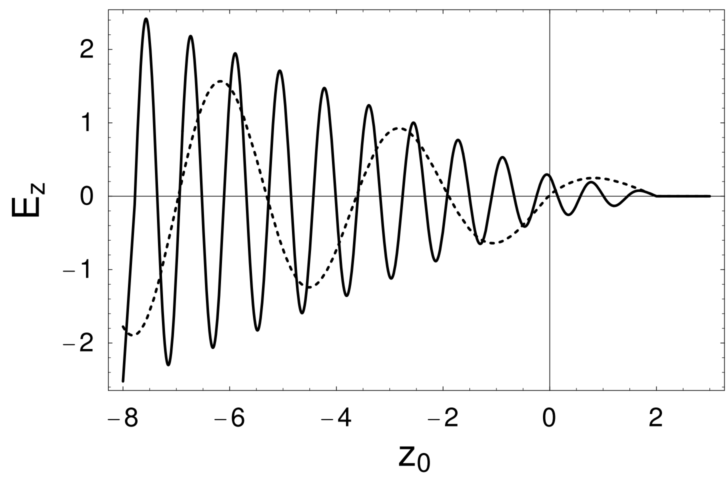

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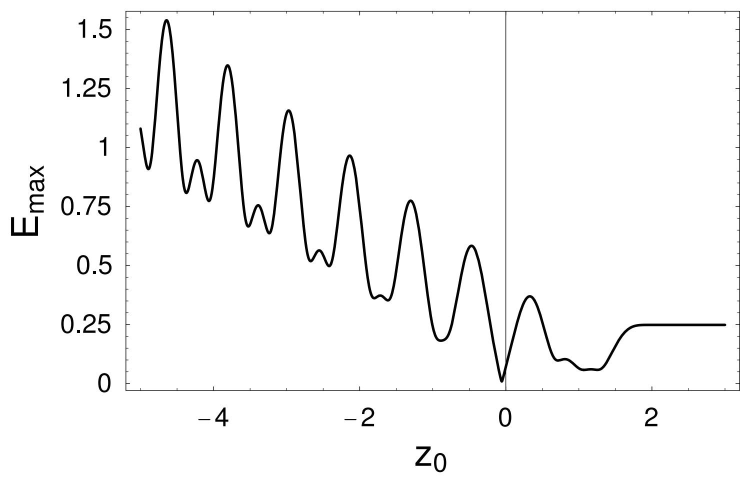

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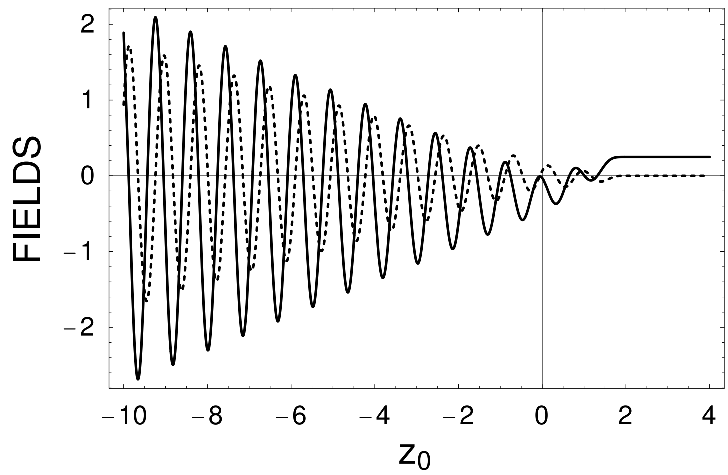

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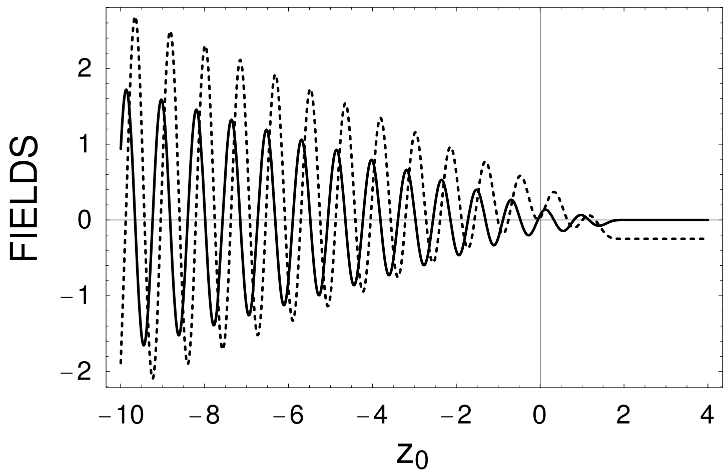
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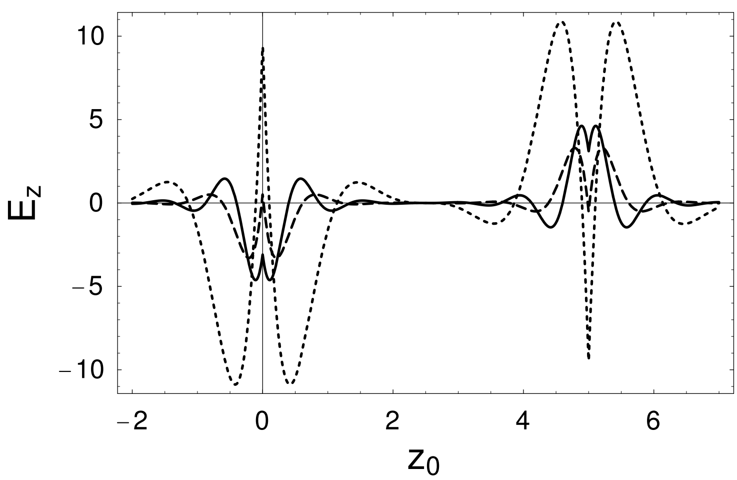

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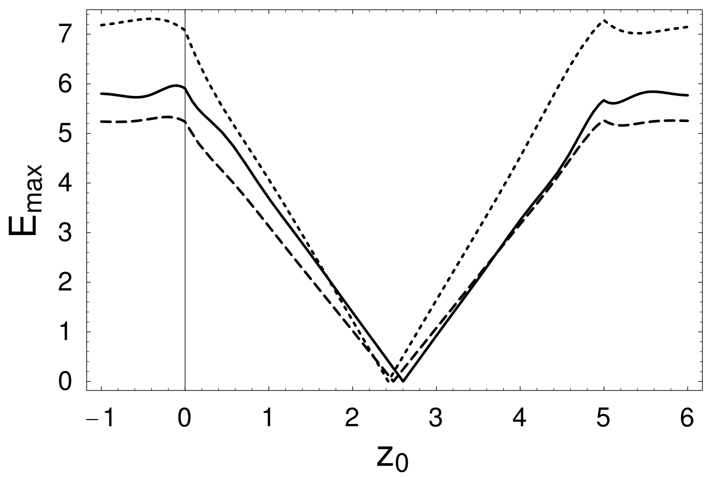

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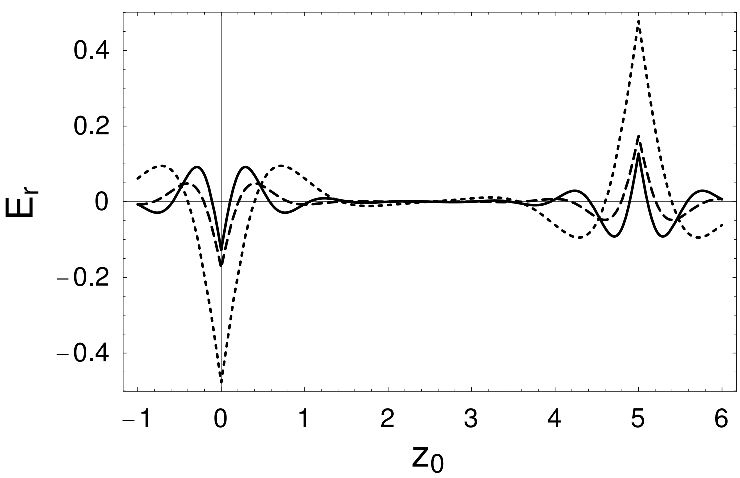

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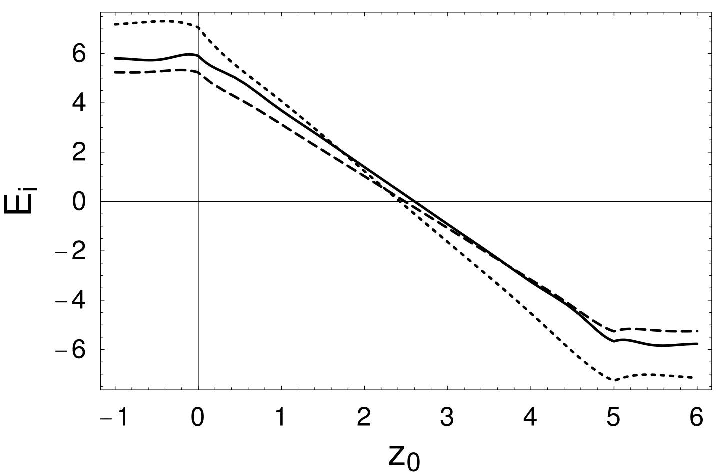

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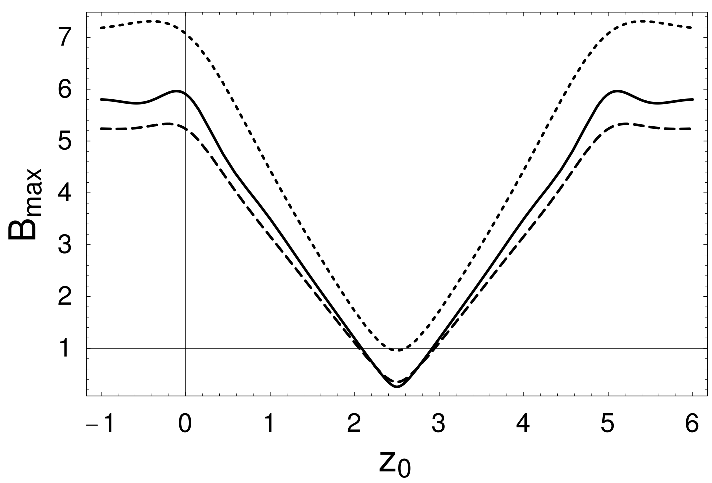

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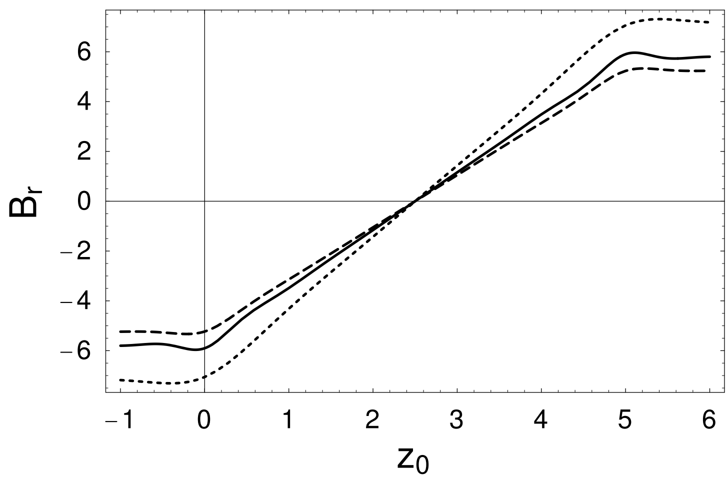

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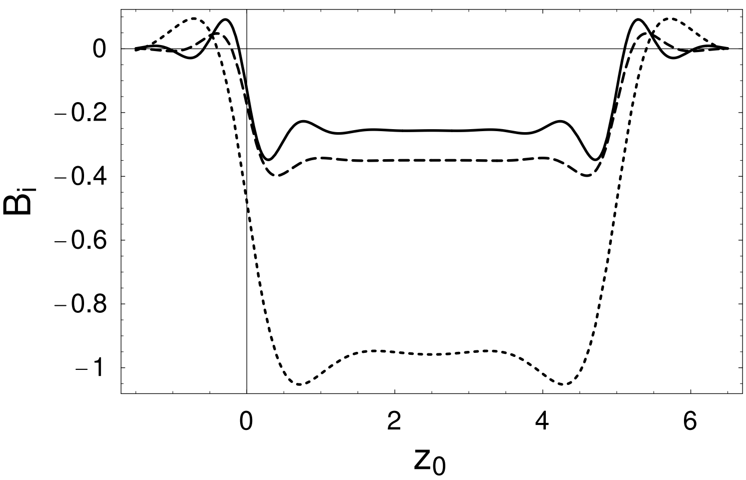

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